A NEW MULTISCALE ANALYSIS OF THE GRADIENT OF LINEAR POLARISATION

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INTRODUCTION
Synchrotron radiation carries the imprint of the magnetic field at the point of origin and along the propagation path.

**Linear polarisation vector**

\[ \mathbf{P} = Q + iU \]

**Amplitude of polarisation**

\[ |\mathbf{P}| = \sqrt{Q^2 + U^2} \]

**Angle of polarisation**

\[ \theta = \left(\frac{1}{2}\right)\tan^{-1}(U/Q) \]

**Faraday rotation**

\[ \theta = \theta_0 + \phi \lambda^2 \]

**Faraday depth**

\[ \phi = 0.81 \int_{\text{source}}^{\text{observer}} n_e \mathbf{B} \cdot d\mathbf{l} \]
THE GRADIENT OF LINEAR POLARISATION
LETTER

Low-Mach-number turbulence in interstellar gas revealed by radio polarization gradients

B. M. Gaensler¹, M. Havenskorn², B. Burkhart³, K. J. Newton–McGee¹, R. D. Ekers⁶, A. Lazarian⁵, N. M. McClure–Griffiths⁶, T. Robishaw⁴, J. M. Dickey⁷ & A. J. Green¹
Gaensler et al. 2011

\[ |\nabla \mathbf{P}| = \sqrt{\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2} \]

\(|\nabla \mathbf{P}|\) measures the rate at which the polarisation vector traces out a trajectory in the Q–U plane as a function of position on the sky.
THE GRADIENT OF LINEAR POLARISATION
**Figure**

- Smoothed and unsmoothed signal
- Derivative of the unsmoothed signal
  - May enhance noise in the data (Burkhart, Lazarian & Gaensler 2012)
- Derivative of the smoothed signal
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Why should we smooth maps before calculating the gradient?


**Figure:** The spatial gradient of linearly polarized emission, $|\nabla P|$, at the original resolution (left) and for the smoothed Stokes Q and U maps (right). Smoothed maps are produced with a convoluted Gaussian filter having a standard deviation of $2^2$ pixels. (Canadian Galactic Plane Survey (CGPS) polarised emission at 1420 MHz – Dr. Roland Kothes’s poster)
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Convolution

\[ \frac{\partial}{\partial x} \]
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- The direct convolution of the Derivative of a Gaussian (DoG)
- The function satisfies the properties of a wavelet transform
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Wavelet Transform

\[ \tilde{f}(l, x) = \begin{cases} \tilde{f}_1 = \frac{1}{l^2} \int \psi_1 \left[ \frac{(x' - x)}{l} \right] f(x') d^2x' \\ \tilde{f}_2 = \frac{1}{l^2} \int \psi_2 \left[ \frac{(x' - x)}{l} \right] f(x') d^2x', \end{cases} \]

where

\[ \psi_1(x, y) = \frac{\partial^m \phi(x, y)}{\partial x^m} \quad \text{and} \quad \psi_2(x, y) = \frac{\partial^m \phi(x, y)}{\partial y^m}. \]
**Original Gradient of Linear Polarisation:**

\[
|\nabla P| = \sqrt{\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2}
\]

**Multiscaled Gradient of Linear Polarisation:**

\[
|\nabla \tilde{P}(l, x)| = \sqrt{|\tilde{Q}(l, x)|^2 + |\tilde{U}(l, x)|^2},
\]

\[
|\tilde{Q}(l, x)| \quad = \sqrt{|\tilde{Q}_1(l, x)|^2 + |\tilde{Q}_2(l, x)|^2},
\]

\[
|\tilde{U}(l, x)| \quad = \sqrt{|\tilde{U}_1(l, x)|^2 + |\tilde{U}_2(l, x)|^2}.
\]
MULTISCALE ANALYSIS OF THE GRADIENT OF LINEAR POLARISATION

*Figure:* From left to right: the $|\nabla \tilde{P}(l, x)|$ values at four different scales $l = 9.6, 45.7, 153.6$ and $434.4$ arcmin. White lines represent maxima chains corresponding to the scale (WTMM, Arnéodo et al. 2000, European J. Phys. B, *15*, 567).
**Figure:** The superposition of maxima chains from scale \( l = 22.8 \) to 258.3 arcmin over the map of \( |\nabla \tilde{P}| \) at \( l = 22.8 \) arcmin. They represent a subset maxima chains for which the maximum value along the chain is part of outliers separated with the scale-wise CVE algorithm.
POWER SPECTRUM
The power spectrum of an image can be calculated from its wavelet coefficients.

Δ-variance analysis (Stutzki et al. 1998, Bensch et al. 2001, Ossenkopf et al. 2008)

Some directional wavelets can reproduce the classical Fourier power spectrum (Kirby 2005, Robitaille, Joncas & Miville-Deschênes 2014).

\[
\int |f(x)|^2 d^2x = C_{\psi}^{-1} \int \int \frac{|\tilde{f}(l, x)|^2}{l^2} \, dl d^2x \tag{1}
\]

\[
E(l) = \int \frac{|\tilde{f}(l, x)|^2}{l^2} \, d^2x \tag{2}
\]

\[
S_P(l) = \frac{1}{N_x N_y} \sum_x |\nabla \tilde{P}(l, x)|^2 \tag{3}
\]
Comparison between the wavelet power spectrum of $|\nabla \tilde{P}|$ and the Fourier power spectrum of $|P|$
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CONCLUSION
It is possible to extend the calculation of $|\nabla \tilde{P}|$ to multiple scales using a wavelet analysis formalism.

Fluctuations traced by $|\nabla \tilde{P}|$ exist at larger scales on data completed with lower spatial frequencies.

We can measure the power spectrum of $|\nabla \tilde{P}|$ using the wavelet formalism.

This analysis will be applied to different radio polarisation data (S-PASS) and the algorithm adapted for calculation on a sphere.

Some features ("large-scale" double jumps, coherent features across scales, ...) must be compared with simulations.