A NEW MULTISCALE ANALYSIS OF THE GRADIENT OF LINEAR POLARISATION

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Magnetic Fields in the Universe V
Cargèse, Corsica

INTRODUCTION

Synchrotron radiation carries the imprint of the magnetic field at the point of origin and along the propagation path.

Linear polarisation vector	$\mathrm{P} = \mathrm{Q} + i\mathrm{U}$
Amplitude of polarisation	$\lvert\mathbf{P}\rvert = \sqrt{\mathrm{Q}^2 + \mathrm{U}^2}$
Angle of polarisation	$\theta = (1/2)\tan^{-1}(\mathrm{U}/\mathrm{Q})$
Faraday rotation	$\theta = \theta_0 + \phi\lambda^2$
Faraday depth	$\phi = 0.81 \int_{\mathrm{source}}^{\mathrm{observer}} n_e \mathbf{B} \cdot dl$

THE GRADIENT OF LINEAR POLARISATION

THE GRADIENT OF LINEAR POLARISATION

LETTER

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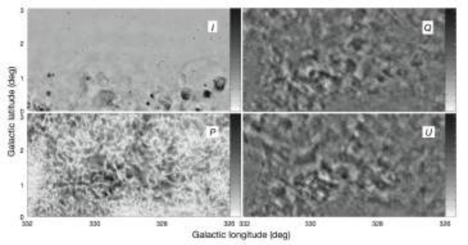
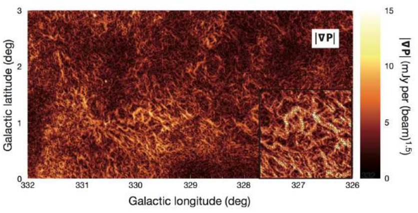
Low-Mach-number turbulence in interstellar gas revealed by radio polarization gradients

B. M. Gaensler[1], M. Haverkorn[2,3,4], B. Burkhart[5], K. J. Newton-McGee[1,6], R. D. Ekers[6], A. Lazarian[5], N. M. McClure-Griffiths[6], T. Robishaw[1], J. M. Dickey[7] & A. J. Green[1]

THE GRADIENT OF LINEAR POLARISATION

Gaensler et al. 2011

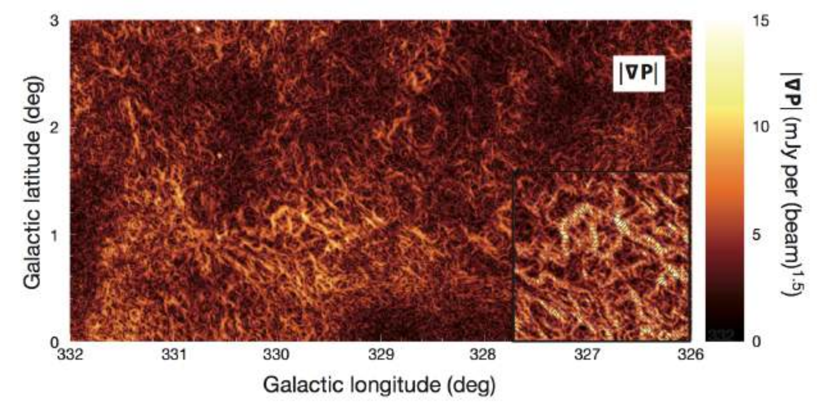




$$|\nabla P| = \sqrt{\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2}$$

$|\nabla P|$ measures the rate at which the polarisation vector traces out a trajectory in the Q–U plane as a function of position on the sky.

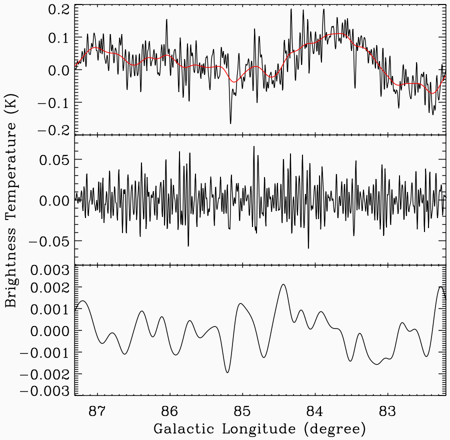
THE GRADIENT OF LINEAR POLARISATION



Figure

- Smoothed and unsmoothed signal
- Derivative of the unsmoothed signal
  - May enhance noise in the data (Burkhart, Lazarian & Gaensler 2012)
- Derivative of the smoothed signal

Robitaille & Scaife 2015 MNRAS, **451 (1)**, 4891



MULTISCALE ANALYSIS OF THE GRADIENT OF LINEAR POLARISATION

Why should we smooth maps before calculating the gradient?

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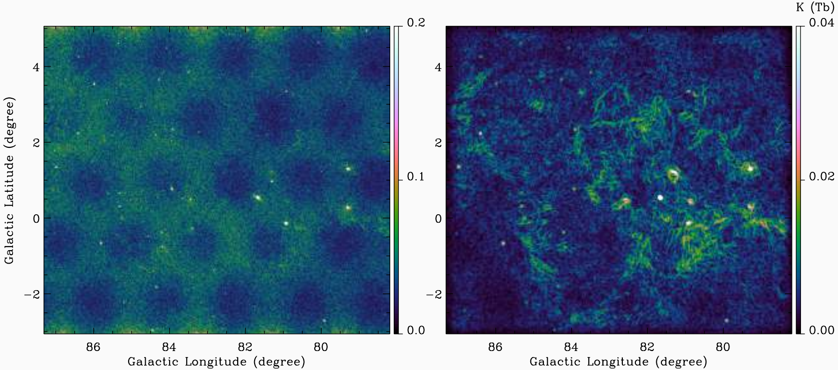
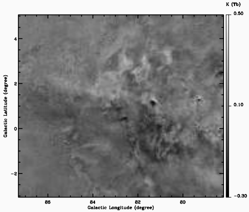
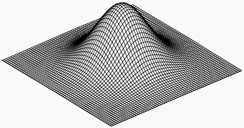


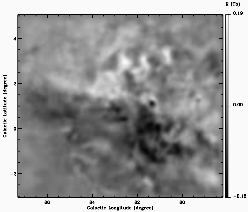
Figure: The spatial gradient of linearly polarized emission, $|\nabla P|$, at the original resolution (left) and for the smoothed Stokes Q and U maps (right). Smoothed maps are produced with a convoluted Gaussian filter having a standard deviation of 2^2 pixels. (Canadian Galactic Plane Survey (CGPS) polarised emission at 1420 MHz (Landecker et al. 2010 – Dr. Roland Kothes's poster)

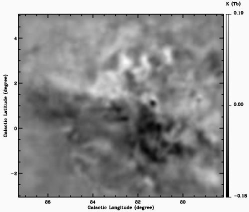
MULTISCALE ANALYSIS OF THE GRADIENT OF LINEAR POLARISATION



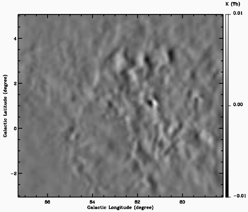
Convolution

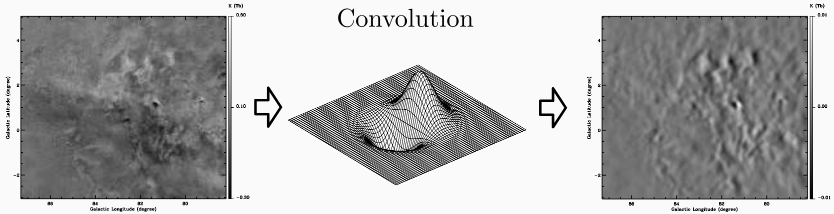






$$\frac{\partial}{\partial x}$$





- The direct convolution of the Derivative of a Gaussian (DoG)
- The function satisfies the properties of a wavelet transform

Wavelet Transform

$$\tilde{f}(l, \mathbf{x}) = \begin{cases} \tilde{f}_1 = \frac{1}{l^2} \int \psi_1 \left[\frac{(\mathbf{x}' - \mathbf{x})}{l} \right] f(\mathbf{x}') d^2\mathbf{x}' \\ \\ \tilde{f}_2 = \frac{1}{l^2} \int \psi_2 \left[\frac{(\mathbf{x}' - \mathbf{x})}{l} \right] f(\mathbf{x}') d^2\mathbf{x}', \end{cases}$$

where

$$\psi_1(x, y) = \frac{\partial^m \phi(x, y)}{\partial x^m} \qquad \text{and} \qquad \psi_2(x, y) = \frac{\partial^m \phi(x, y)}{\partial y^m}.$$

MULTISCALE ANALYSIS OF THE GRADIENT OF LINEAR POLARISATION

Original Gradient of Linear Polarisation:

$$|\nabla P| = \sqrt{\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2}$$

Multiscaled Gradient of Linear Polarisation:

$$|\nabla \tilde{P}(l, x)| = \sqrt{|\tilde{Q}(l, x)|^2 + |\tilde{U}(l, x)|^2},$$

$$\begin{aligned} |\tilde{Q}(l, x)| &= \sqrt{|\tilde{Q}_1(l, x)|^2 + |\tilde{Q}_2(l, x)|^2}, \\ |\tilde{U}(l, x)| &= \sqrt{|\tilde{U}_1(l, x)|^2 + |\tilde{U}_2(l, x)|^2}. \end{aligned}$$

MULTISCALE ANALYSIS OF THE GRADIENT OF LINEAR POLARISATION

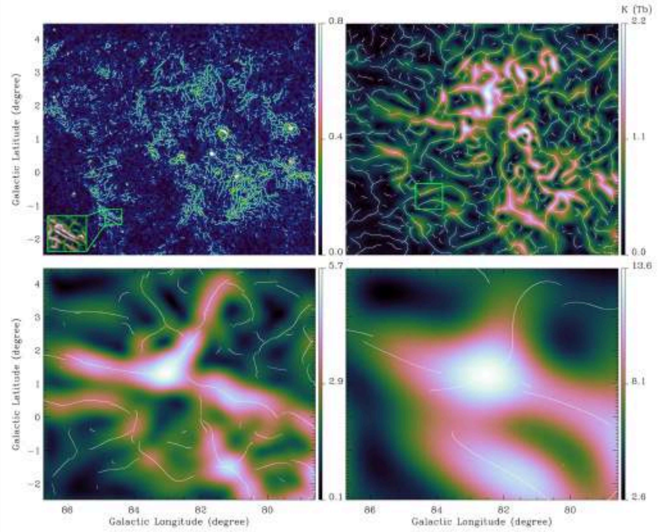


Figure: From left to right: the $|\nabla\tilde{P}(l, x)|$ values at four different scales l= 9.6, 45.7, 153.6 and 434.4 arcmin. White lines represent maxima chains corresponding to the scale (WTMM, Arnéodo et al. 2000, European J. Phys. B, **15**, 567).

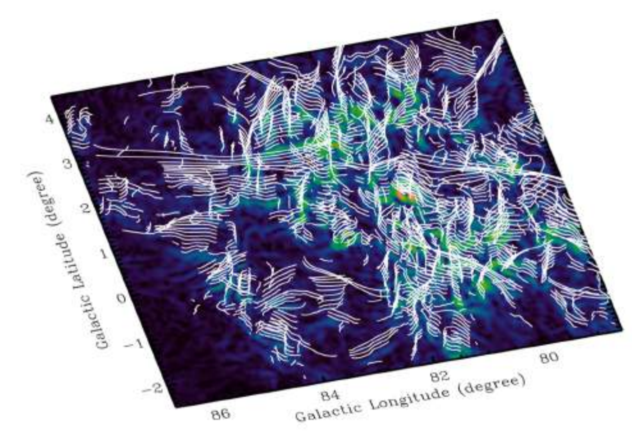


Figure: The superposition of maxima chains from scale l =22.8 to 258.3 arcmin over the map of $|\nabla\tilde{P}|$ at $l = 22.8$ arcmin. They represent a subset maxima chains for which the maximum value along the chain is part of outliers separated with the scale-wise CVE algorithm.

POWER SPECTRUM

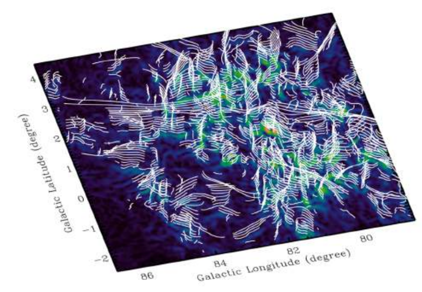
- The power spectrum of an image can be calculated from its wavelet coefficients.
- Δ-variance analysis (Stutzki et al. 1998, Bensch et al. 2001, Ossenkopf et al. 2008)
- Some directional wavelets can reproduce the classical Fourier power spectrum (Kirby 2005, Robitaille, Joncas & Miville-Deschênes 2014).

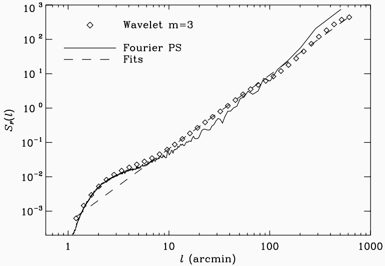
$$\int |f(\mathbf{x})|^2 d^2\mathbf{x} = C_\psi^{-1} \int \int \frac{|\tilde{f}(l, \mathbf{x})|^2}{l^2} dl d^2\mathbf{x} \tag{1}$$

$$E(l) = \int \frac{|\tilde{f}(l, \mathbf{x})|^2}{l^2} d^2\mathbf{x} \tag{2}$$

$$S_P(l) = \frac{1}{N_x N_y} \sum_{\mathbf{x}} |\nabla \tilde{P}(l, \mathbf{x})|^2 \tag{3}$$

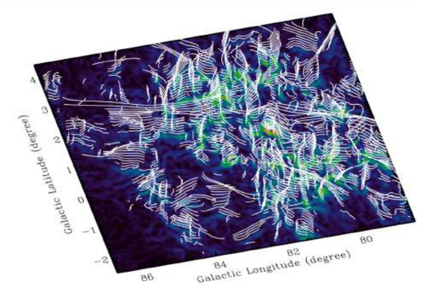
Comparison between the wavelet power spectrum of $|\nabla\tilde{P}|$ and the Fourier power spectrum of $|P|$

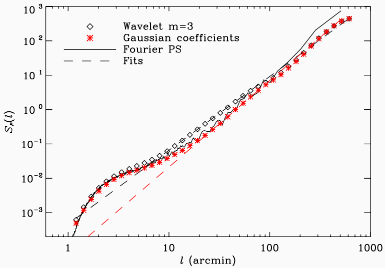




POWER SPECTRUM

Comparison between the wavelet power spectrum of $|\nabla \tilde{P}|$ and the Fourier power spectrum of $|P|$





Comparison between the wavelet power spectrum of $|\nabla\tilde{\mathrm{P}}|$ and the Fourier power spectrum of $|\mathrm{P}|$

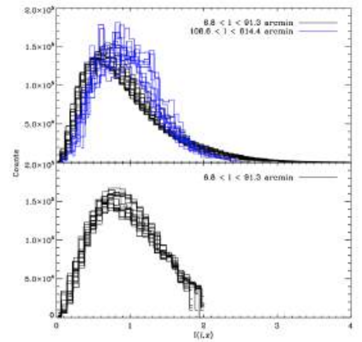
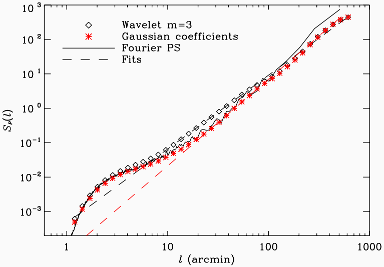


Figure 12. Normalised distributions of wavelet coefficients of $|\nabla\tilde{P}(l,x)|$ for the CGPS field for all coefficients (top panel). The black lines represent scales between 6.8 and 91.3 arcmin and the blue lines present scales between 108.6 and 614.4 arcmin. The lower panel shows the Gaussian part of the distribution for scales between 6.8 and 91.3 arcmin.



CONCLUSION

- It is possible to extend the calculation of $|\nabla\tilde{P}|$ to multiple scales using a wavelet analysis formalism.
- Fluctuations traced by $|\nabla\tilde{P}|$ exist at larger scales on data completed with lower spatial frequencies.
- We can measure the power spectrum of $|\nabla\tilde{P}|$ using the wavelet formalism.
- This analysis will be applied to different radio polarisation data (S-PASS) and the algorithm adapted for calculation on a sphere.
- Some features ("large-scale" double jumps, coherent features across scales, ...) must be compared with simulations.