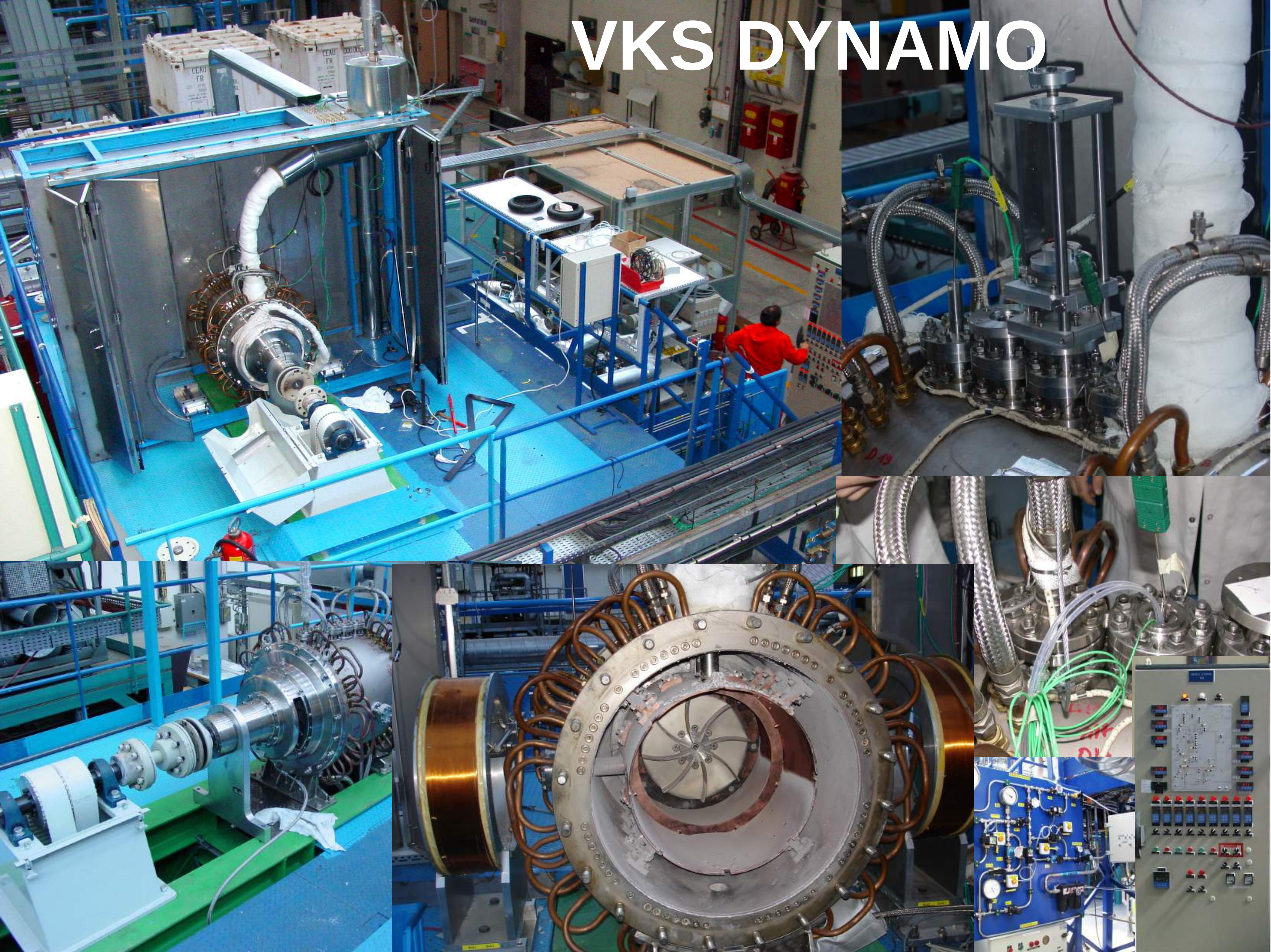
Magnetic fluid-structure Dynamo ?

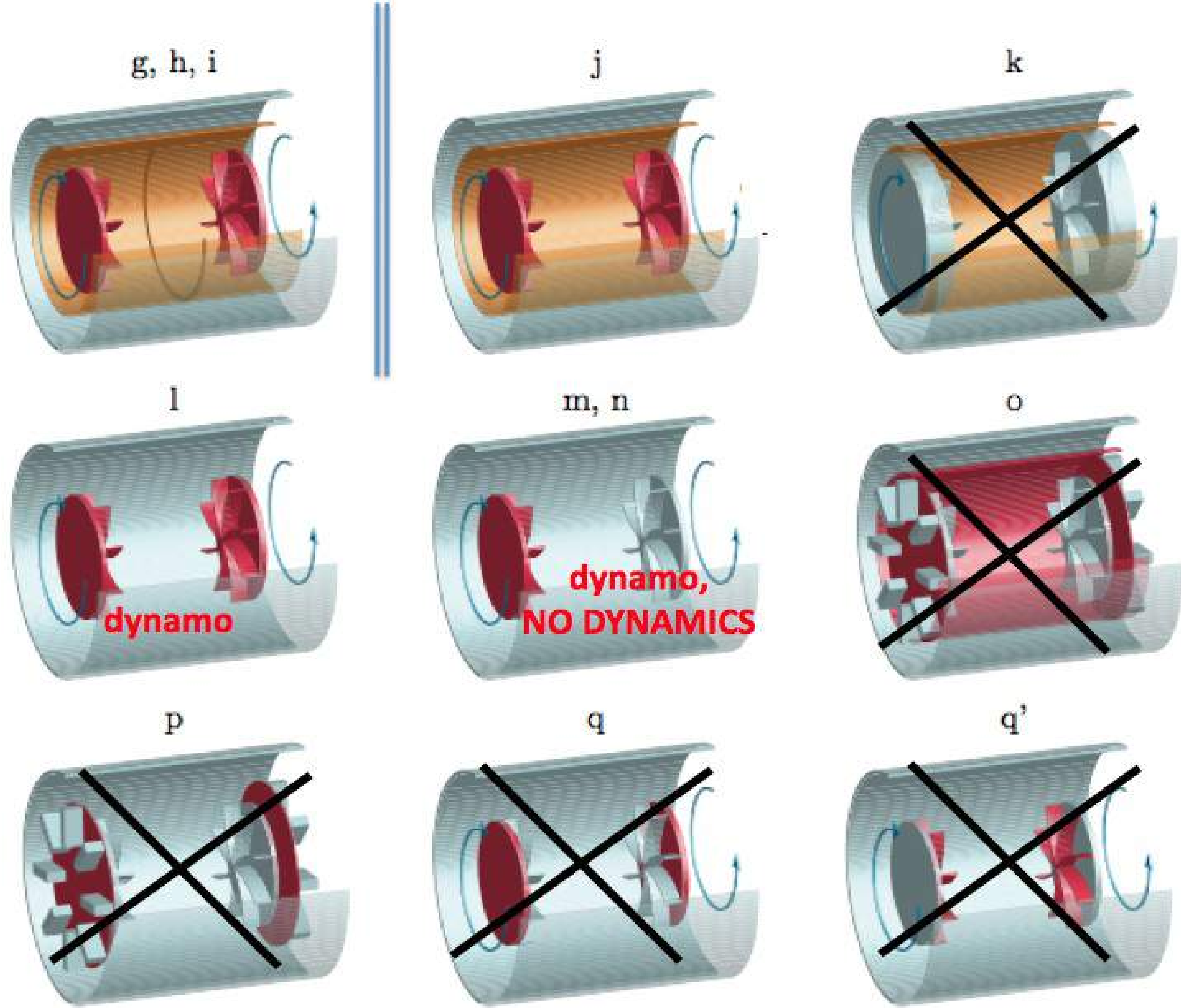
Von Kármán dynamo via a volume penalization method

S. Kreuzahler , Y. Ponty , H. Homann, N. Plihon, R. Grauer

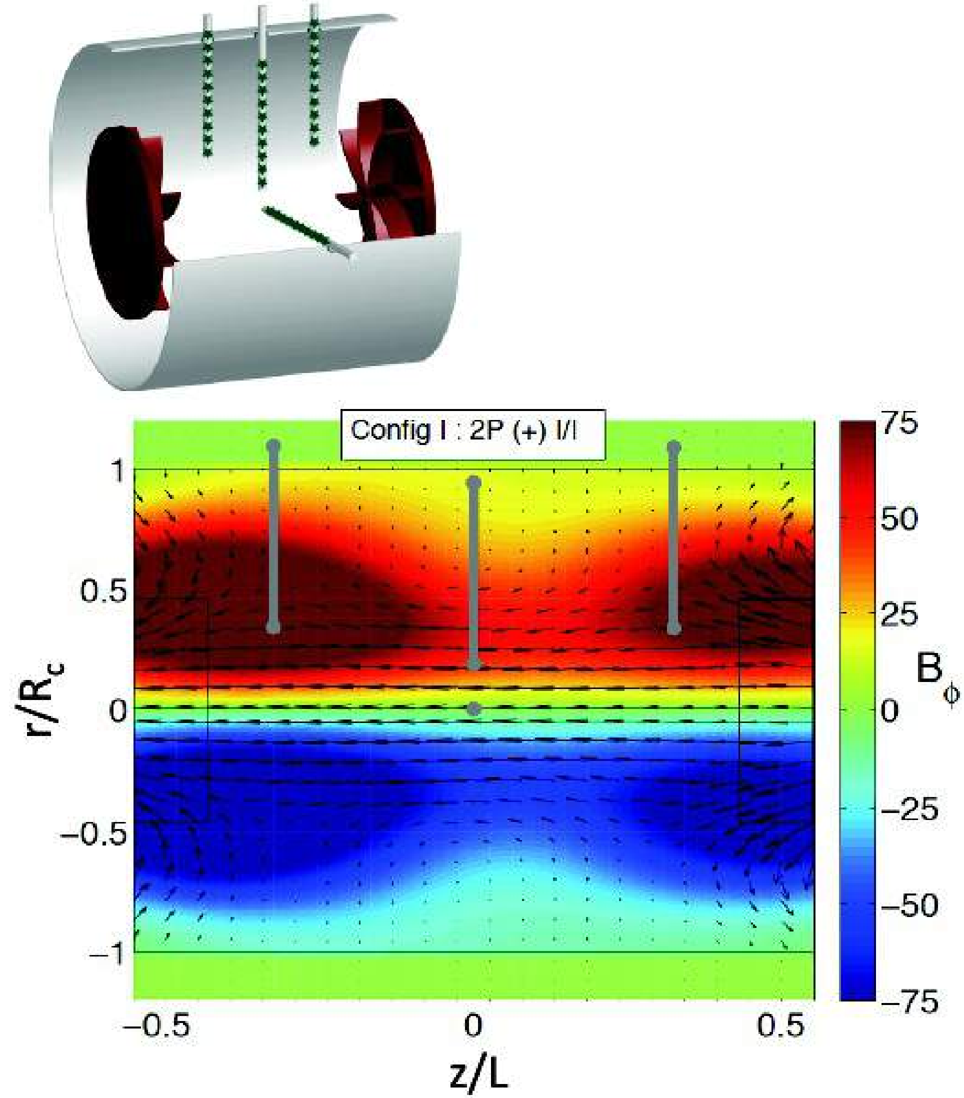
ENS Lyon
Laboratoire de Physique

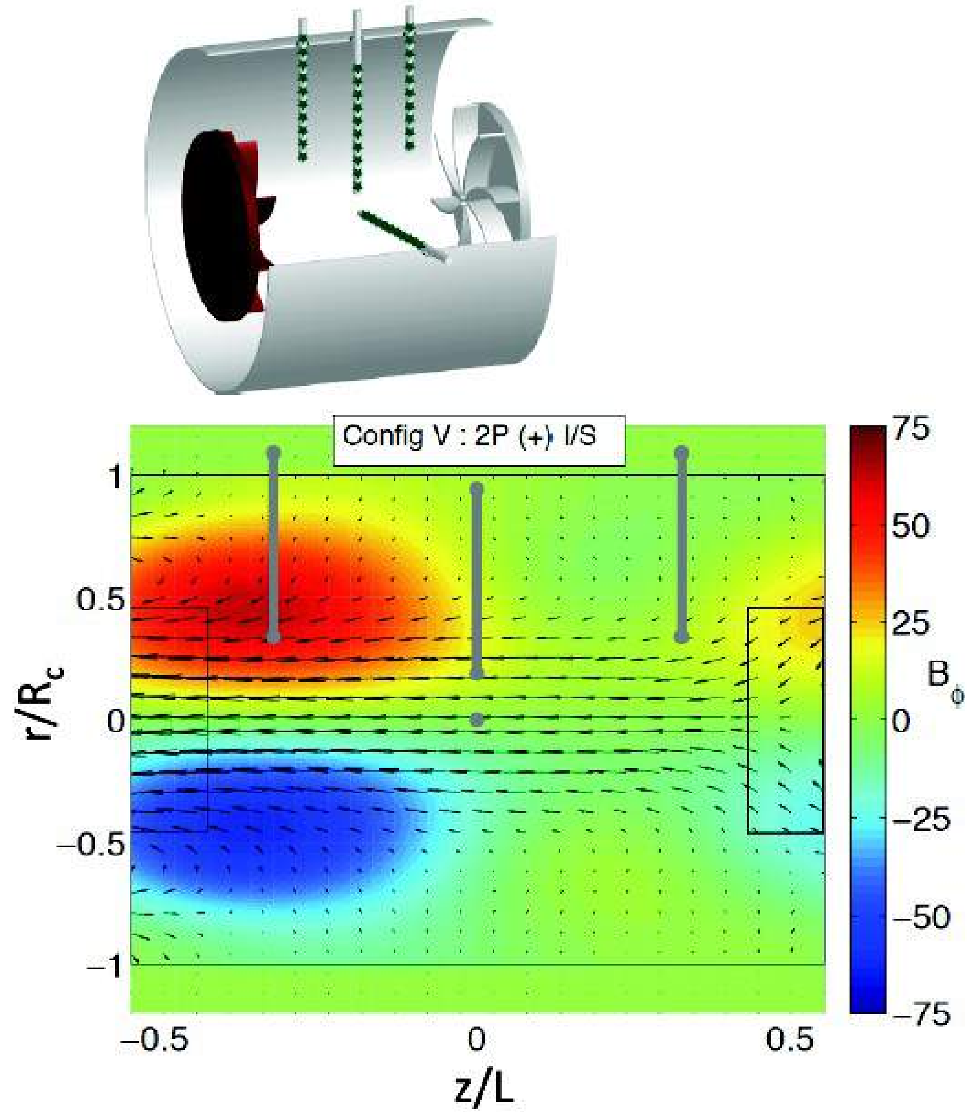
VKS DYNAMO




g, h, i
j
k
l
dynamo
m, n
dynamo,
NO DYNAMICS
o
p
q
q'

Magnetic field reconstruction – Dynamo decomposition



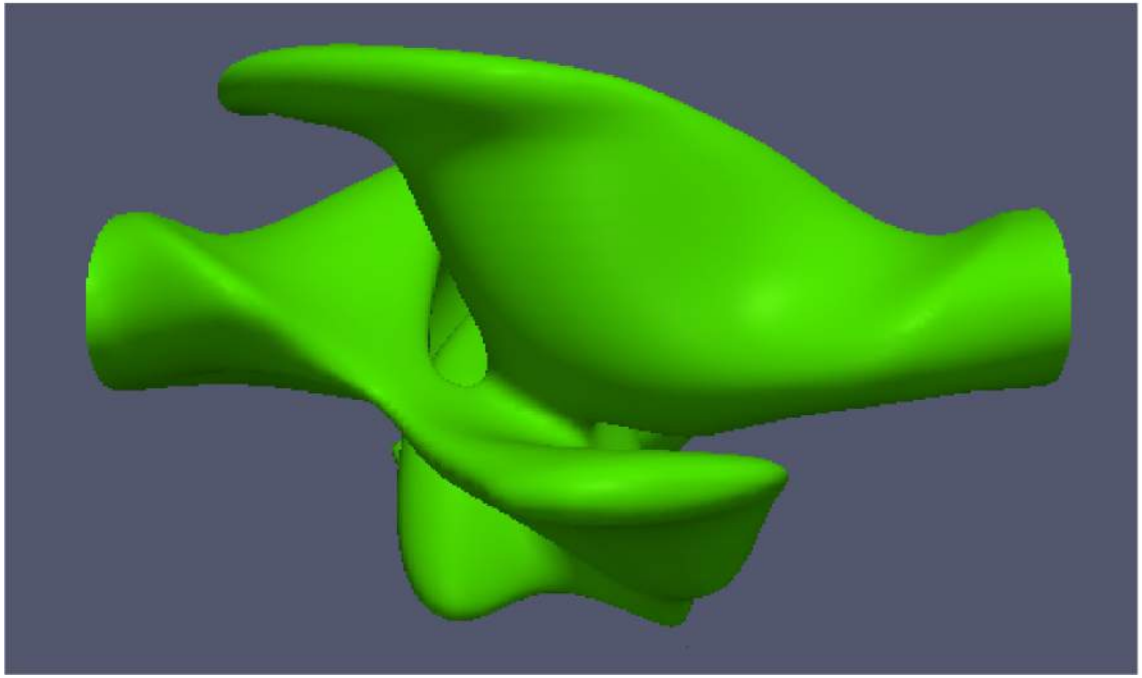


$$B_{II} = \alpha B_{Iss} + \beta r_{\pi}(B_{Iss}) + B_c$$

$$\alpha \approx 1\,;\ \beta \approx -1$$

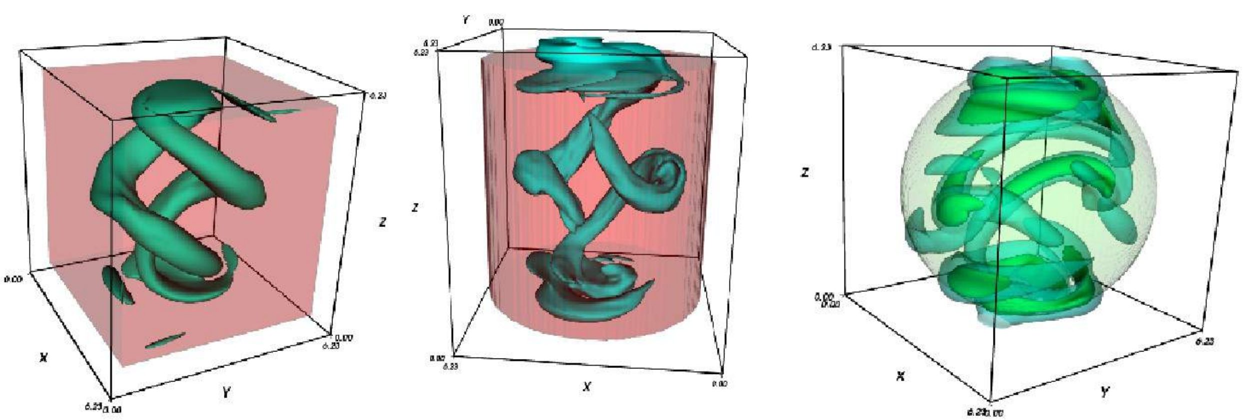
Boisson *et al.*
NJP **14** (2012)

Numerical Von Kármán (VKN)

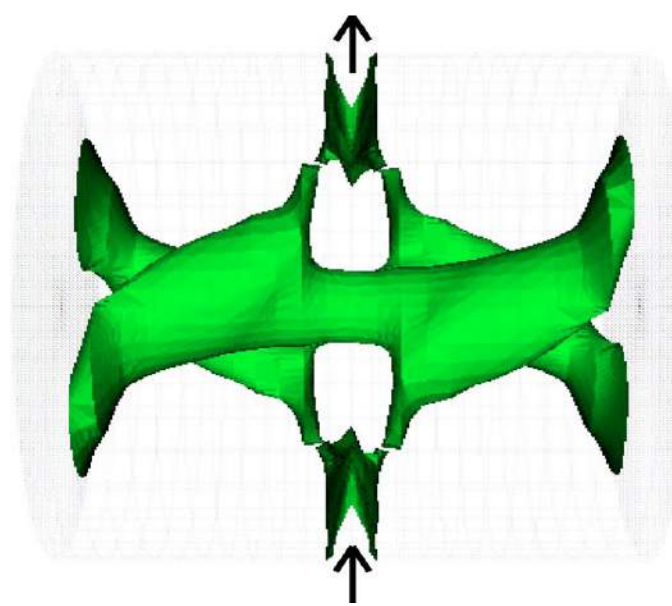


Periodic boundaries for the magnetic field

Mode dynamo m=1

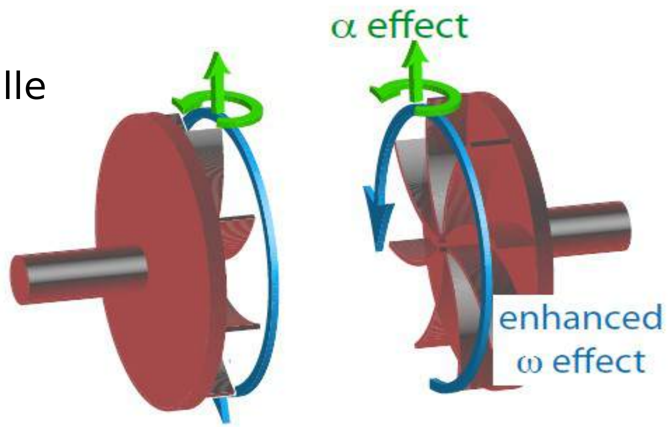


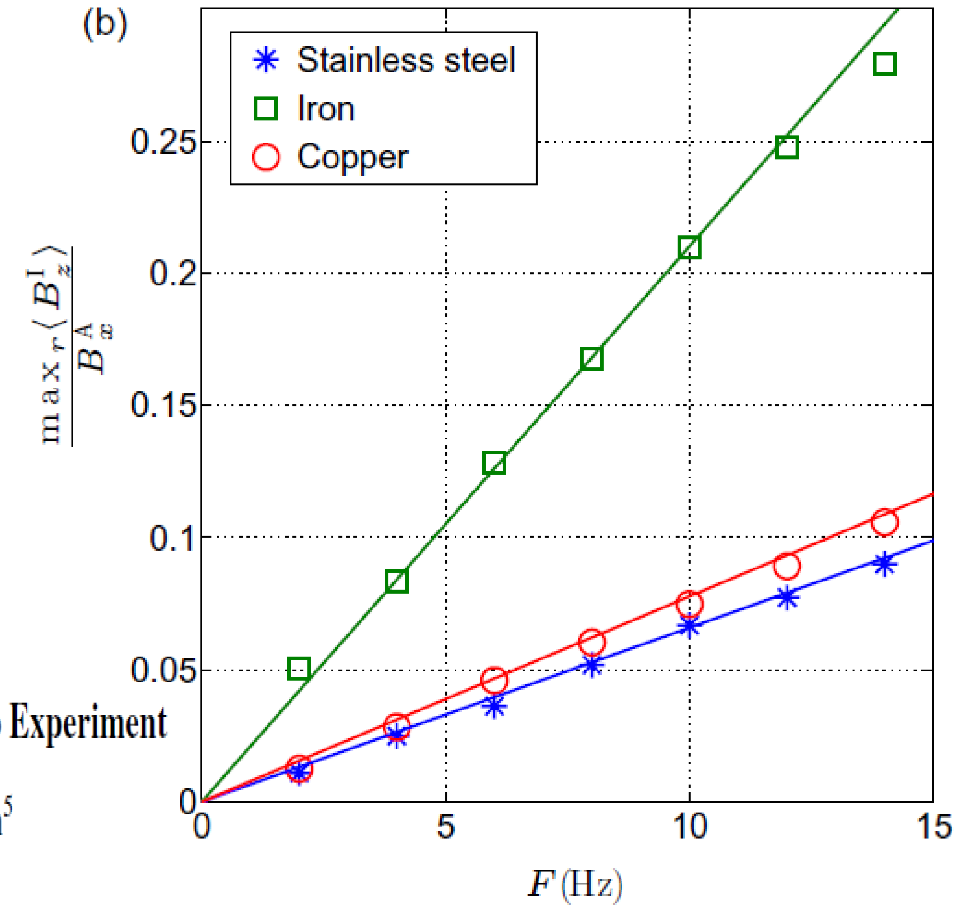
Taylor-Green 1/2



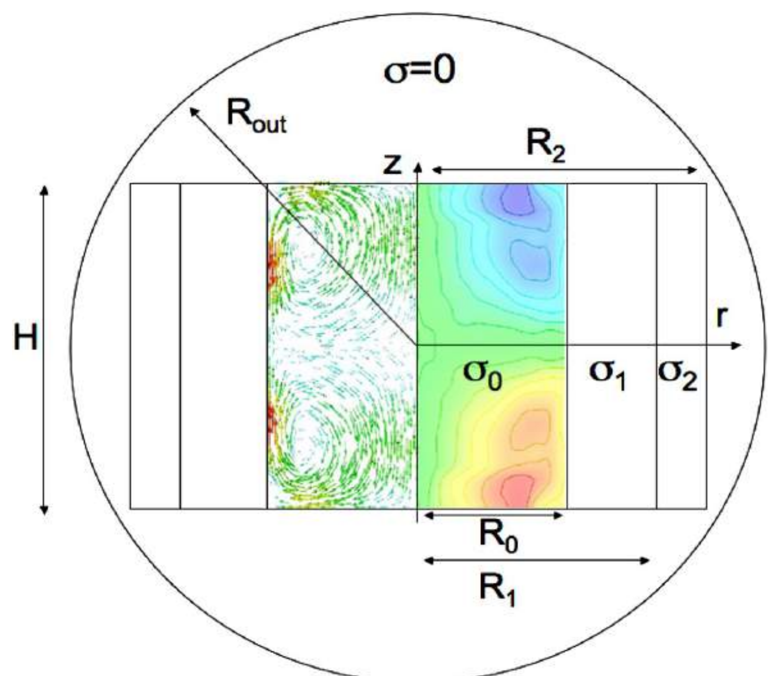
F. Ravelet et al Phys. Fluid 17(11) 2005.

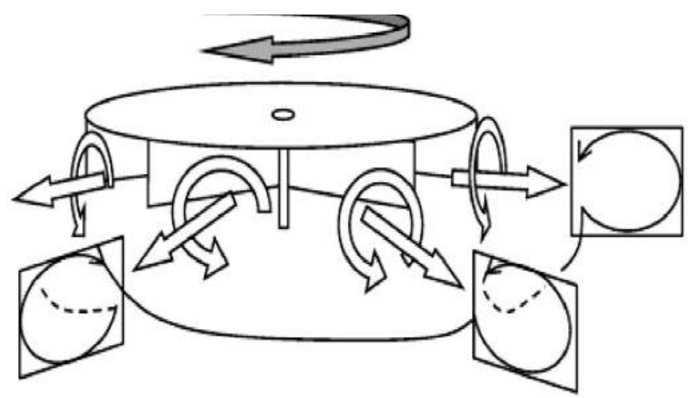
G. Verhille, N. Plihon, M. Bourgoin, P. Odier and J.-F. Pinton
Induction in a von Karman flow driven by ferromagnetic impellers
New J. Phys., 12, 033006 (2010)





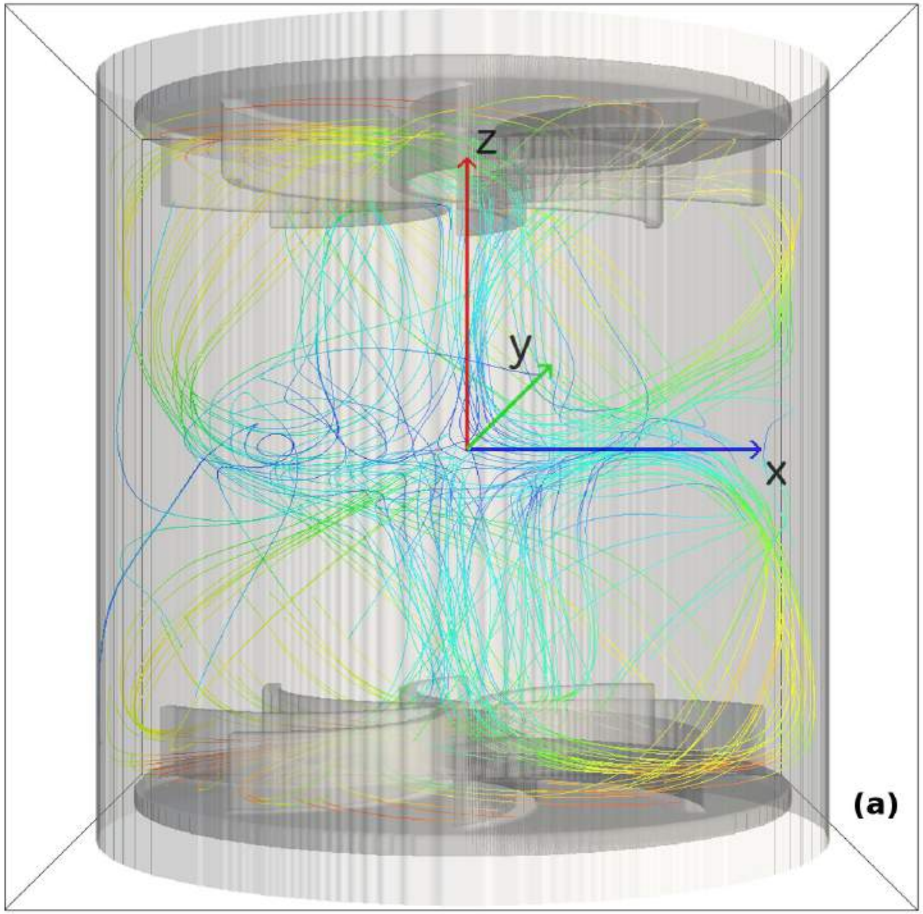
Impact of Impellers on the Axisymmetric Magnetic Mode in the VKS2 Dynamo Experiment
PRL 101, 104501 (2008)
R. Laguerre,[1,2] C. Nore,[2,*] A. Ribeiro,[2] J. Léorat,[3] J.-L. Guermond,[2,4] and F. Plunian[5]





F. Pétrélis, N. Mordant, and S. Fauve, Geophys. Astrophys. Fluid Dyn. **101**, 289 (2007).

Numerical Von Kármán (VKN)



Spectral code : periodic box

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \Delta \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u}|_{boundary} = \mathrm{V}_{penalized}.$$

Moving impellers fix rotation

Cylinder periodic along z

Classic Penalization method

$$\partial_t \vec{u} + \vec{u}\nabla\vec{u} = -\nabla P + \nu\Delta\vec{u} + \vec{j}\times\vec{b} - C\xi(\vec{x})\vec{u}$$

$$\nabla\vec{u} = 0$$

$$\xi(\vec{x}) = 1 \; inside\ boundary$$

$$\xi(\vec{x}) = 0 \; outside\ boundary$$

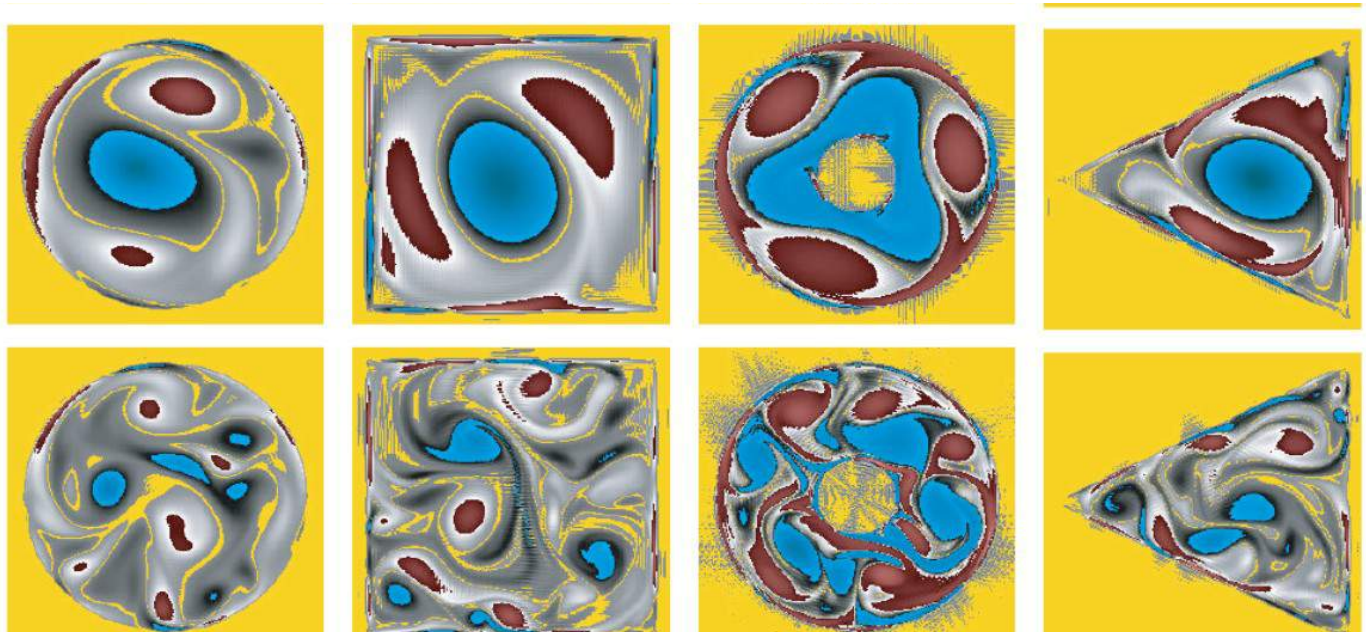
Kai Schneider[1] and Marie Farge[2]

Computational Physics and New Perspectives in Turbulence
Y. Kaneda (Ed.)
Springer, 2007, pp. 241-246

$$\nu\Delta u^{n+1} - \frac{\alpha}{\tau}u^{n+1} - \nabla p^{n+1} = (1-\chi)f^{n+1} \quad \text{in } \Omega,$$

$$\nabla\cdot u^{n+1} = 0,$$

$$B\left(u^{n+1}\right) = g^{n+1} \quad \text{on } \Gamma,$$



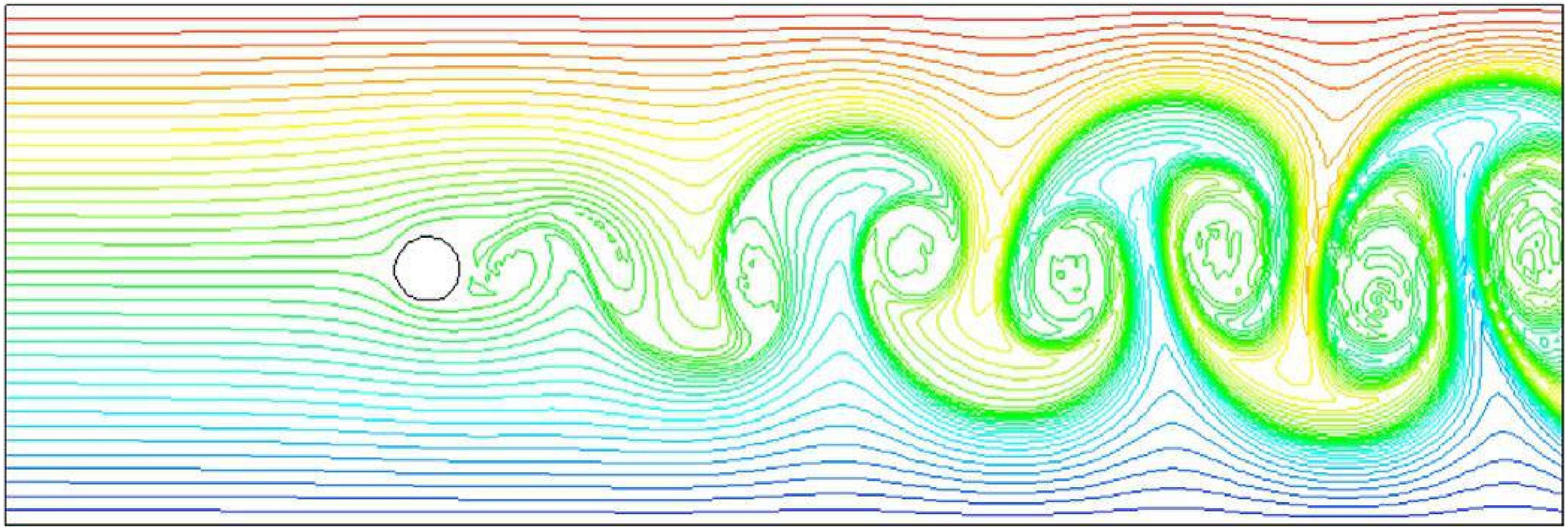
Pseudo-Penalization method

M. Minguez, R. Pasquetti and E. Serre
"A pseudo-penalization method for high Reynolds unsteady flows"
Applied Numerical Mathematics
Volume **58**, Issue 7, **July 2008**, Pages 946-954

$$\nu \Delta u^{n+1} - \frac{\alpha}{\tau} u^{n+1} - \nabla p^{n+1} = (1-\chi) f^{n+1} \quad \text{in } \Omega,$$

$$\nabla \cdot u^{n+1} = 0,$$

$$B\left(u^{n+1}\right) = g^{n+1} \quad \text{on } \Gamma,$$



Penalisation method : Direct forcing

E. A Fadlum et al JCP 161, 35–60 (2000)

$$\mathbf{u}^{(n+1)} = \mathbf{u}^{(n)} + \Delta t \cdot \left(\mathcal{L}\left(\mathbf{u}^{(n)}\right) + \mathbf{f}_b^{(n)} \right)$$

$$\mathbf{f}_b^{(n)} = -\chi_p(\mathbf{r},t)\{\mathcal{L}(\mathbf{u}^{(n)}) + \frac{1}{\Delta t}(\mathbf{u}^{(n)} - \mathbf{V}^{(n+1)})\}$$

J. Mohd-Yusof,
CTR Annual Research Briefs,
NASA Ames/Stanford University, (1997).

Volume Fraction method :

$$\overline{\chi}_p(\mathbf{r}) = \frac{1}{2^3 \Delta x \Delta y \Delta z} \int_{-\Delta x}^{\Delta x} \int_{-\Delta y}^{\Delta y} \int_{-\Delta z}^{\Delta z} \chi_p(\mathbf{r}+\mathbf{r}^*) \mathrm{d}^3 r^*$$

E. A Fadlum et al JCP 161, 35–60 (2000)

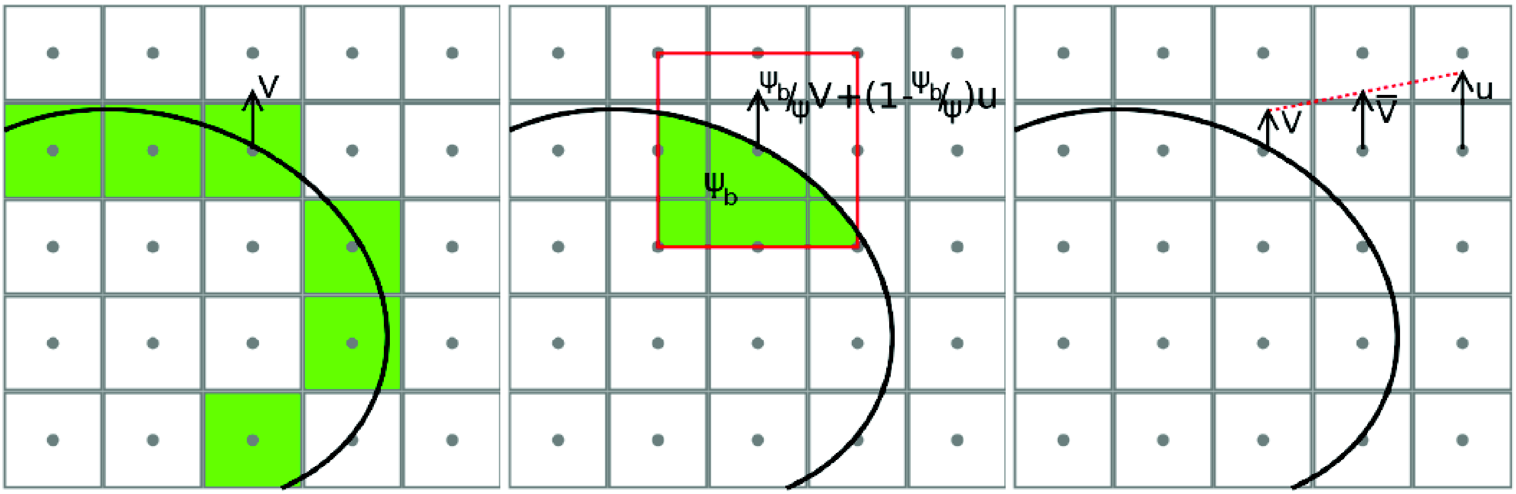


Fig. 3.2.: Stepwise interpolation, volume fraction method and linear interpolation

The pressure predictor

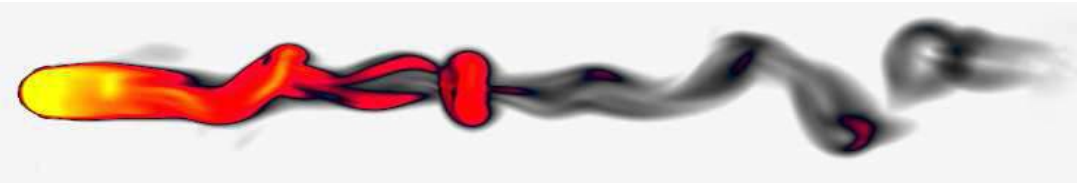
Brown, D. L., Cortez, R. & Minion, M. L. Accurate Projection Methods for the Incompressible Navier Stokes Equations. J. Comp. Phys. 168 (2), 464–499 (2001).

$$\tilde{\tilde{\mathbf{u}}}^{(n+1)} = \mathcal{B}\tilde{\mathbf{u}}^{(n+1)} = \tilde{\mathbf{u}}^{(n+1)} + \mathbf{f}_b^n \ .$$

$$p^n = p^{n-1} + \Phi^n \ , \ \text{where} \ \Delta\Phi^n = \frac{1}{\Delta t}\nabla \cdot \tilde{\tilde{\mathbf{u}}}^{(n+1)} \ .$$

$$\mathbf{u}^{(n+1)} = \mathcal{P}\tilde{\tilde{\mathbf{u}}}^{(n+1)} = \tilde{\tilde{\mathbf{u}}}^{(n+1)} - \nabla p^n$$

H. Homann, J. Bec & R. Grauer JFM 2013.
Effect of turbulent fluctuations on the drag and lift forces on a towed sphere and its boundary layer



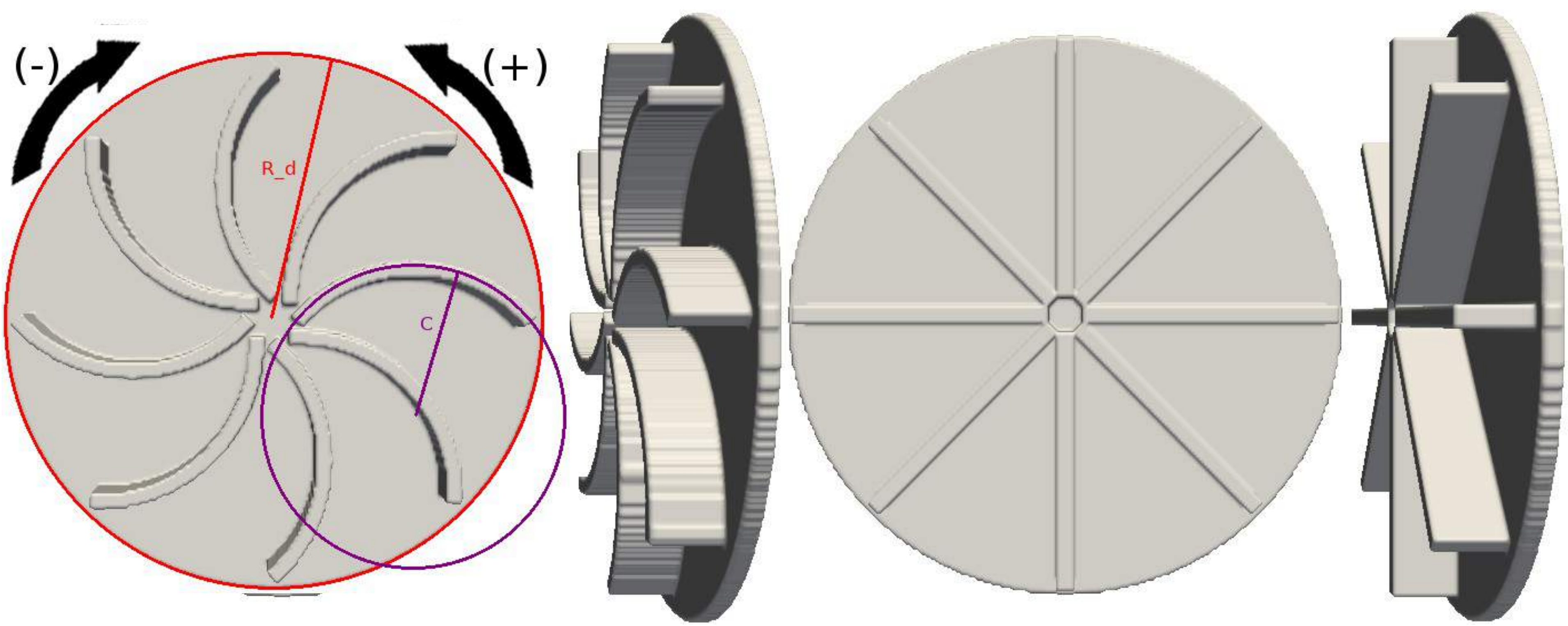
Design of the numerical impellers :

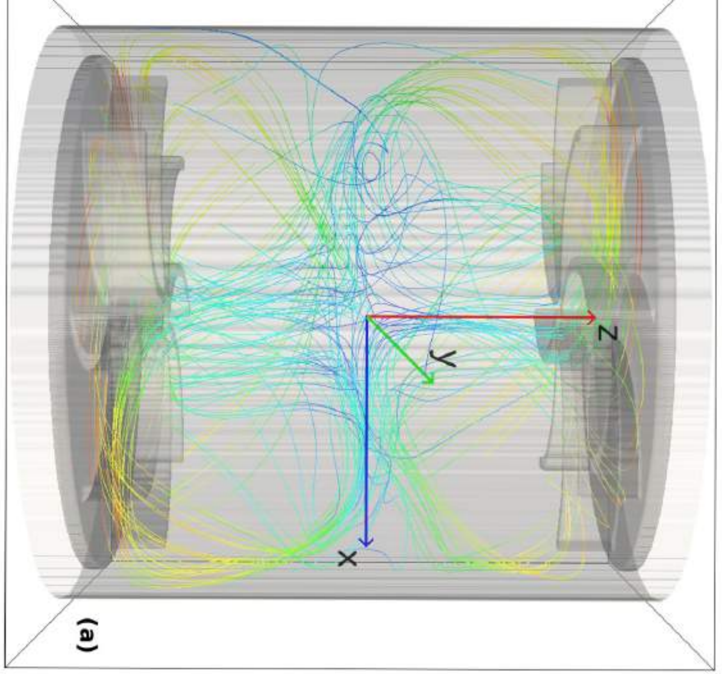
L. Marie et al   EPJB 33(4):469-485, June 2003.

TM28 configuration :

Rc = 3:0 , Rd = 0:9Rc, C = 0:5Rc
height of the eight blades is fix at 0:2Rc

Expulsion angle α = arcsin (Rd=2C) ~ 1:11976 rad ~ 64:15 deg.

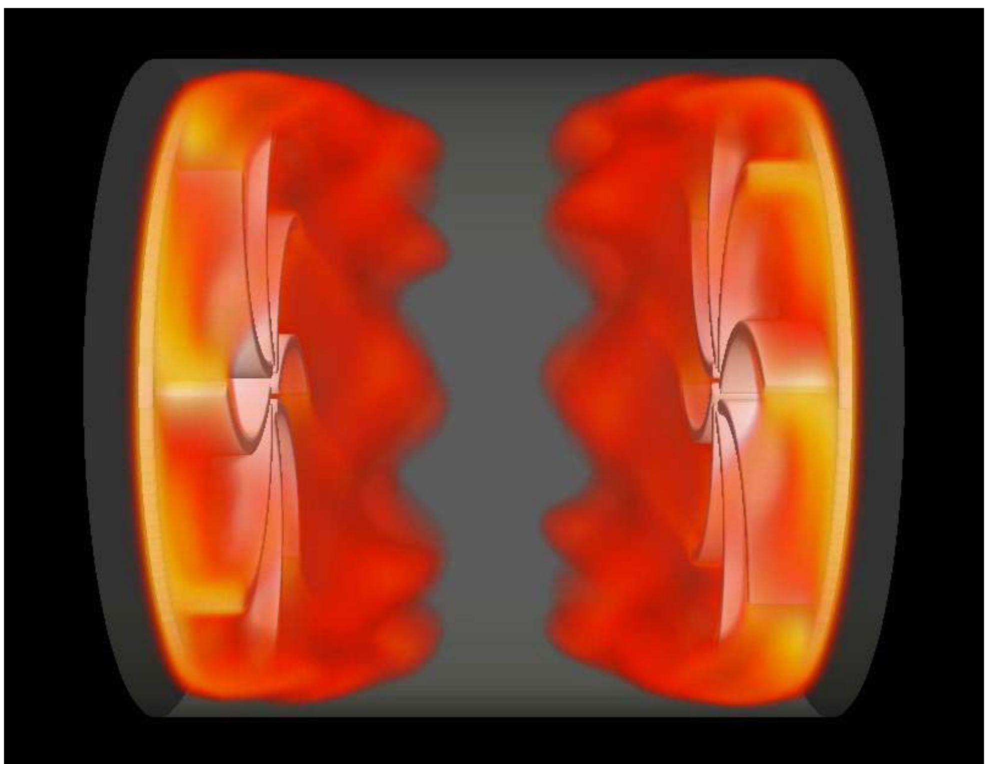




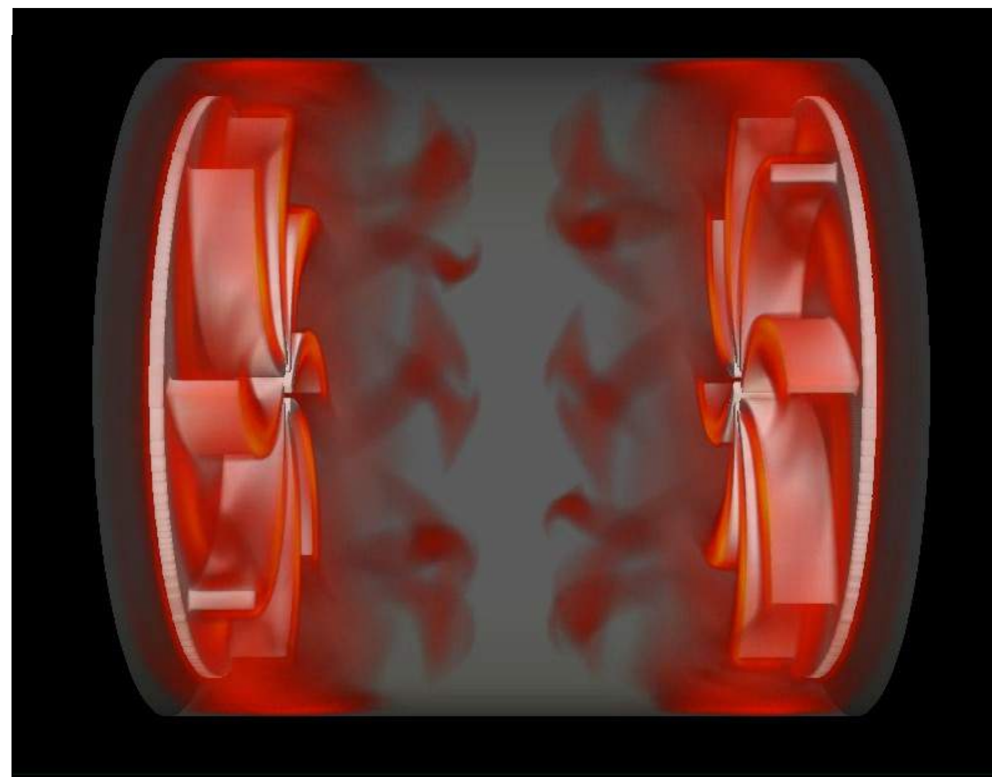
S. Kreuzahler, D. Schulz, H. Homann, Y. Ponty, R. Grauer

"Numerical study of impeller-driven von Karman flows via a volume penalization method"

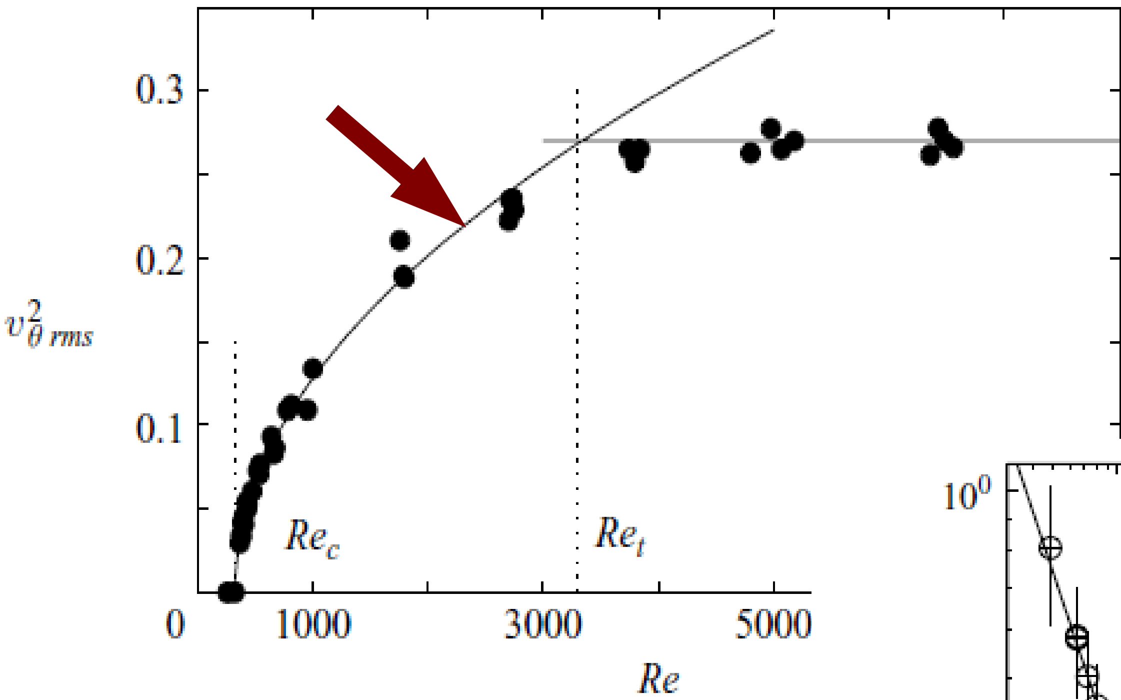
New J. Phys. 16 103001 (2014)
doi:10.1088/1367-2630/16/10/103001

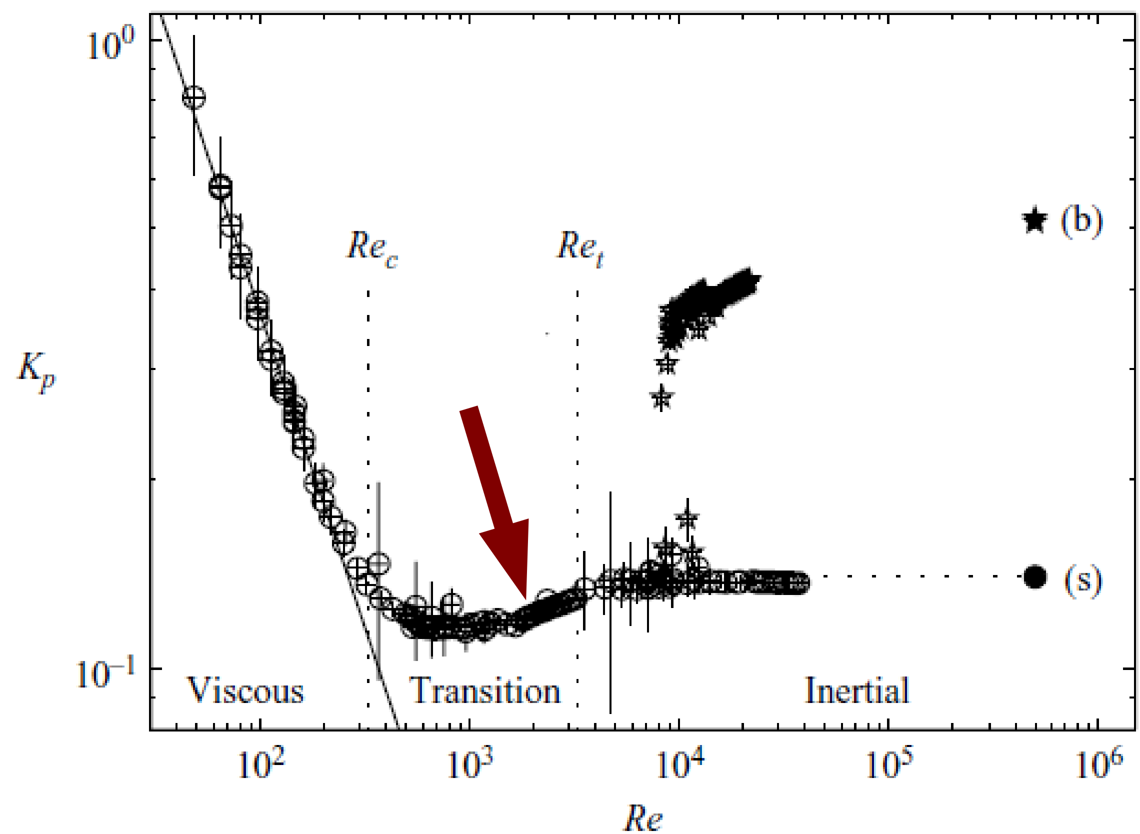


Kinetic Energy



Enstrophy

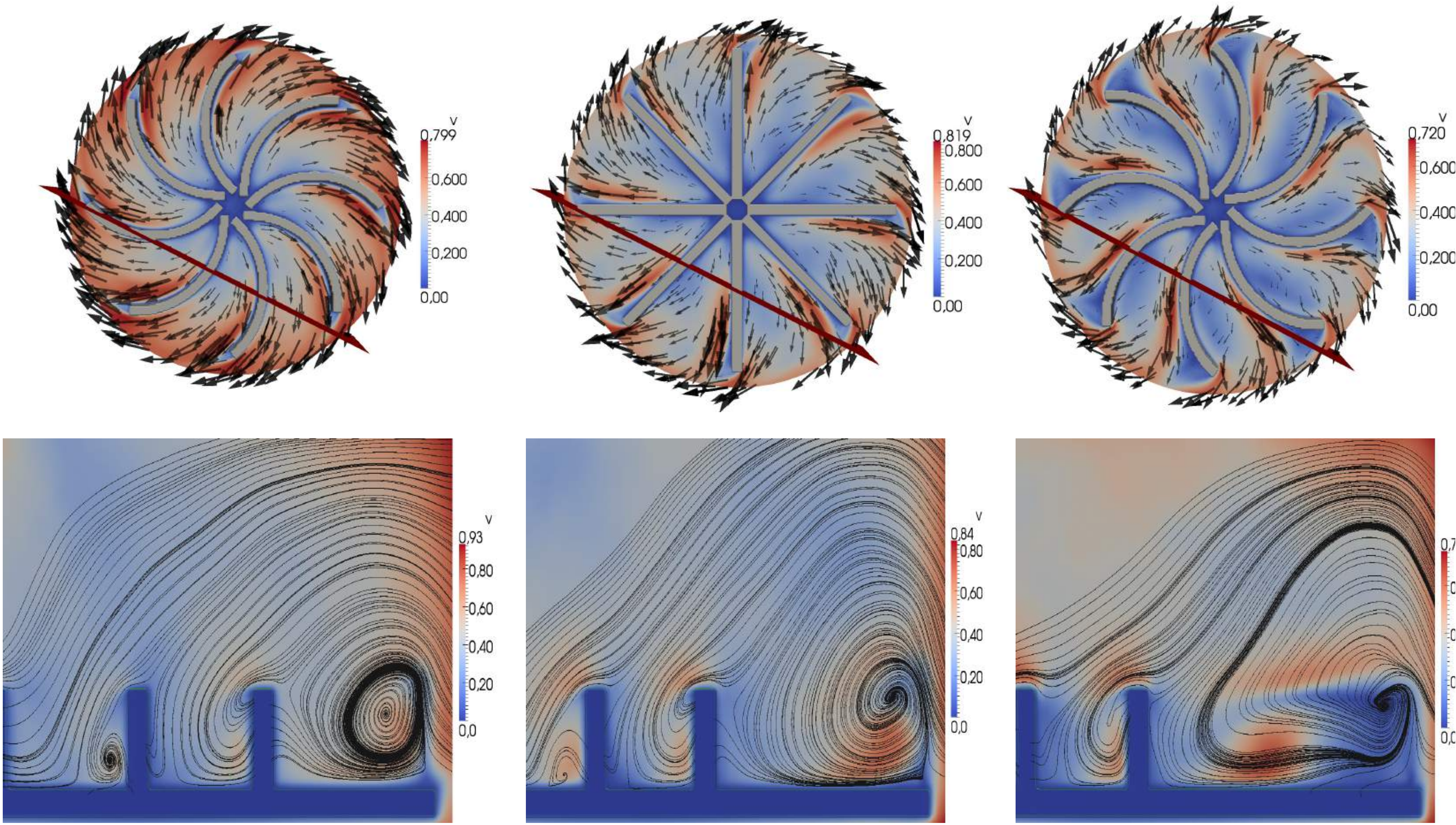




TM60 configuration

Florent Ravelet , Arnaud Chiffaudel and François Daviaud
J. Fluid Mech., vol. 601, pp. 339–364. (2008)

(+) , straight, (-) configurations



Sebastian Kreuzahler, Daniel Schulz, Holger Homann, Yannick Ponty, Rainer Grauer
"Numerical study of impeller-driven von Karman flows via a volume penalization method"
New J. Phys. 16 103001 (2014)

Equation MHD with solid matter

Imcompressible liquid metal + impellers with magnetic permeability mu

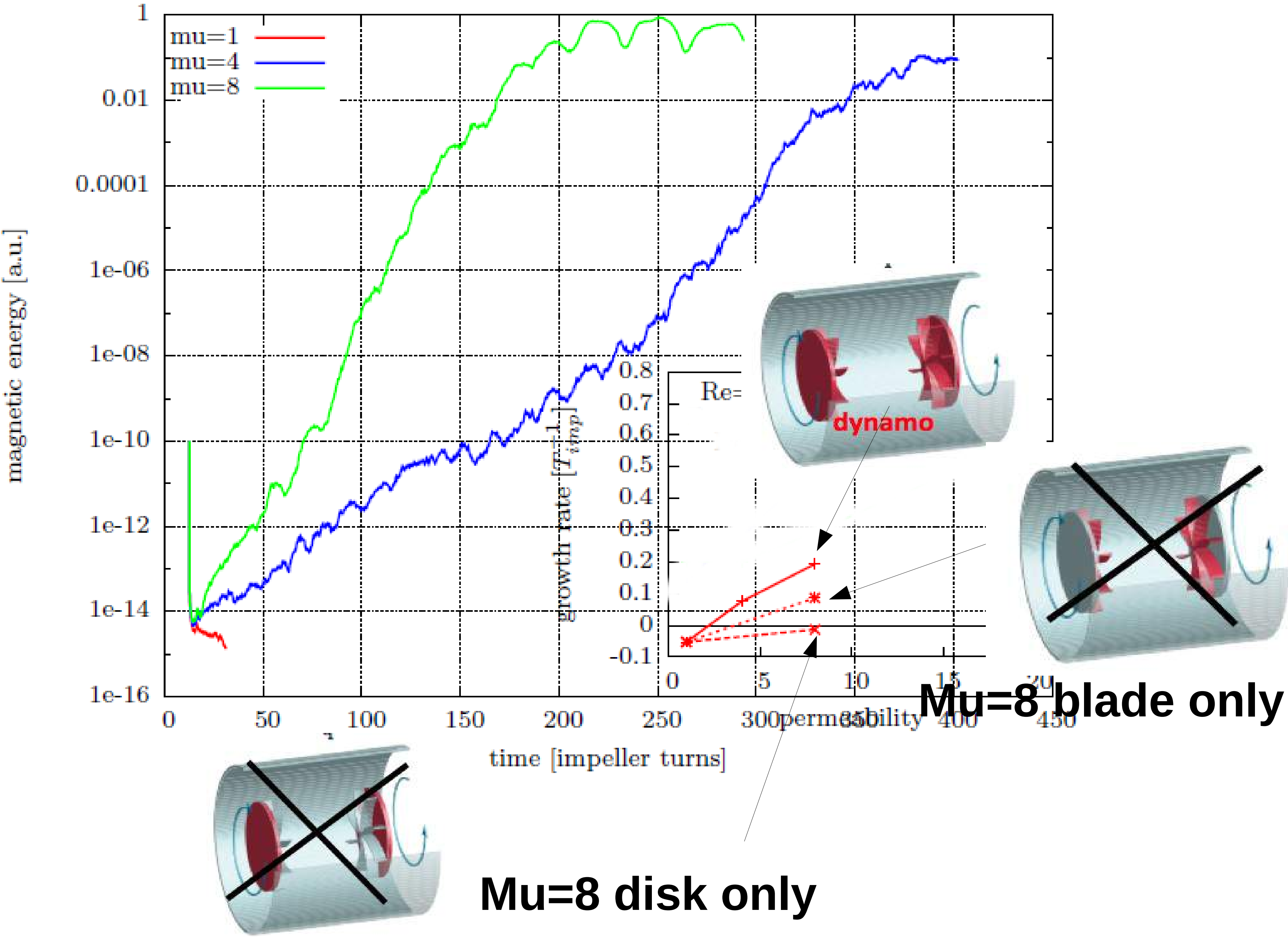
$$\partial_t u + u \nabla u = -\nabla P + (\nabla \times \frac{B}{\mu}) \times B + \nu \nabla^2 u + f_{boundary}$$

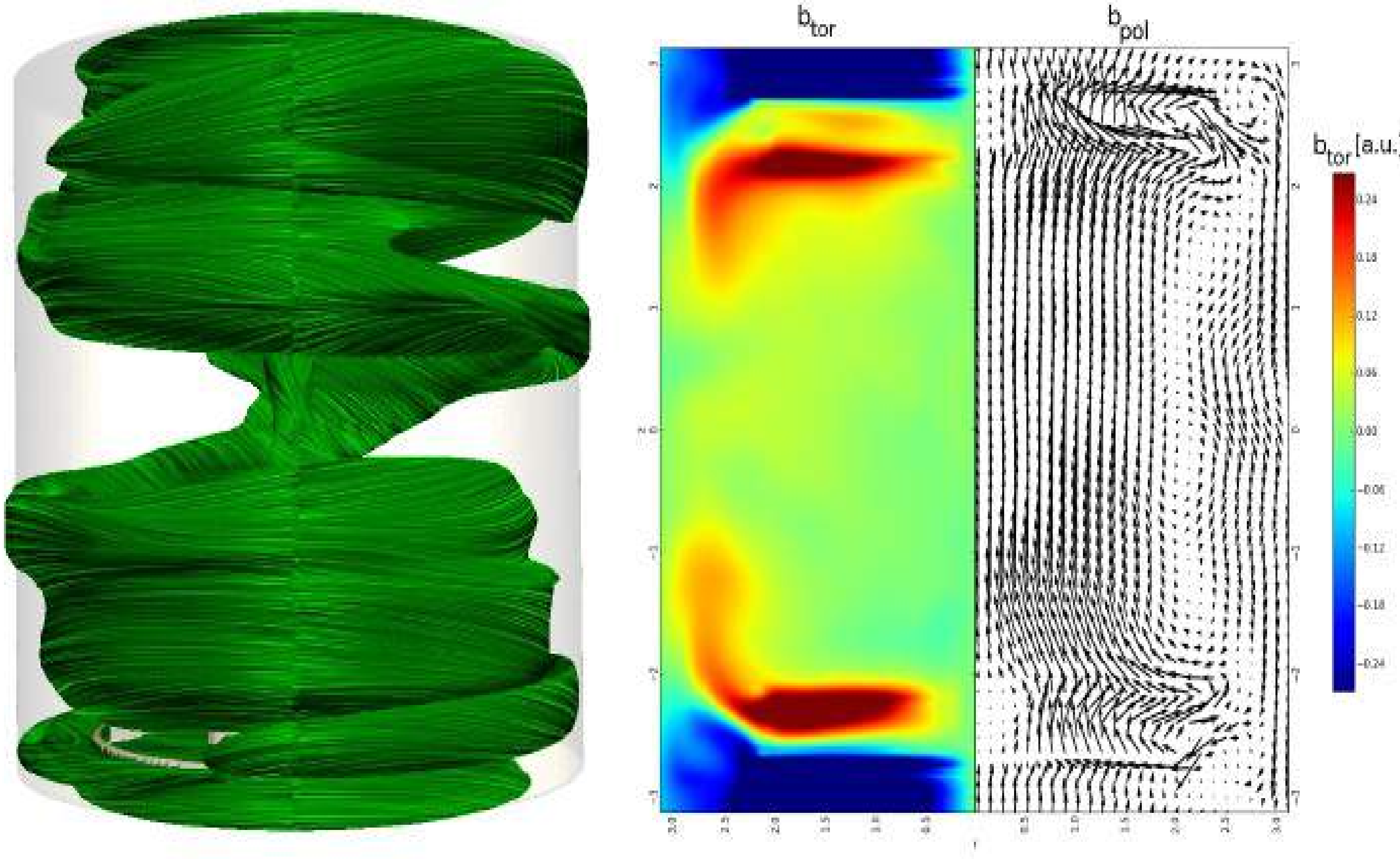
$$\partial_t B = \nabla \times \left(u \times B - \eta \nabla \times \frac{B}{\mu} \right) \qquad \nabla . u = 0 \qquad \nabla . B = 0$$

f_boundary leads to no-slip boundary condition for u

B is free in the periodic box.
Only the jump of magnetic permeability is smoothed by a cosine filter

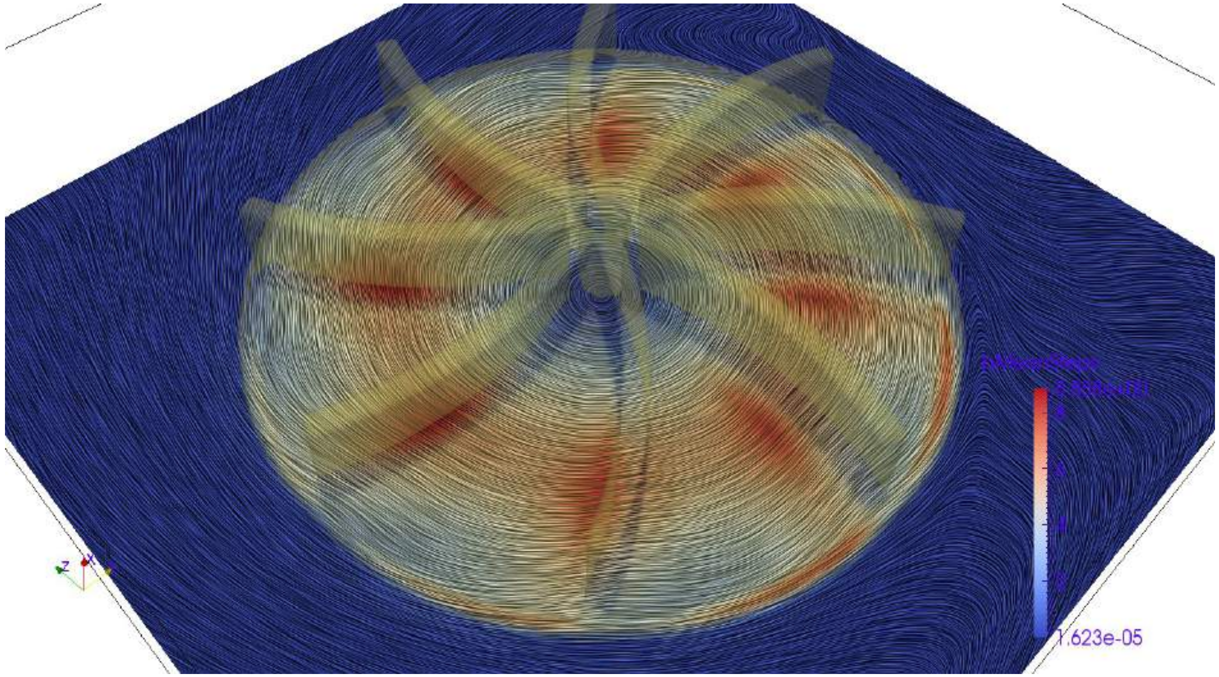
Solved by a pseudo-spectral method with 3 order RK time stepping scheme


mu=1
mu=4
mu=8
magnetic energy [a.u.]
time [impeller turns]
growth rate [T_{imp}^{-1}]
permeability
Re=
dynamo
Mu=8 blade only
Mu=8 disk only



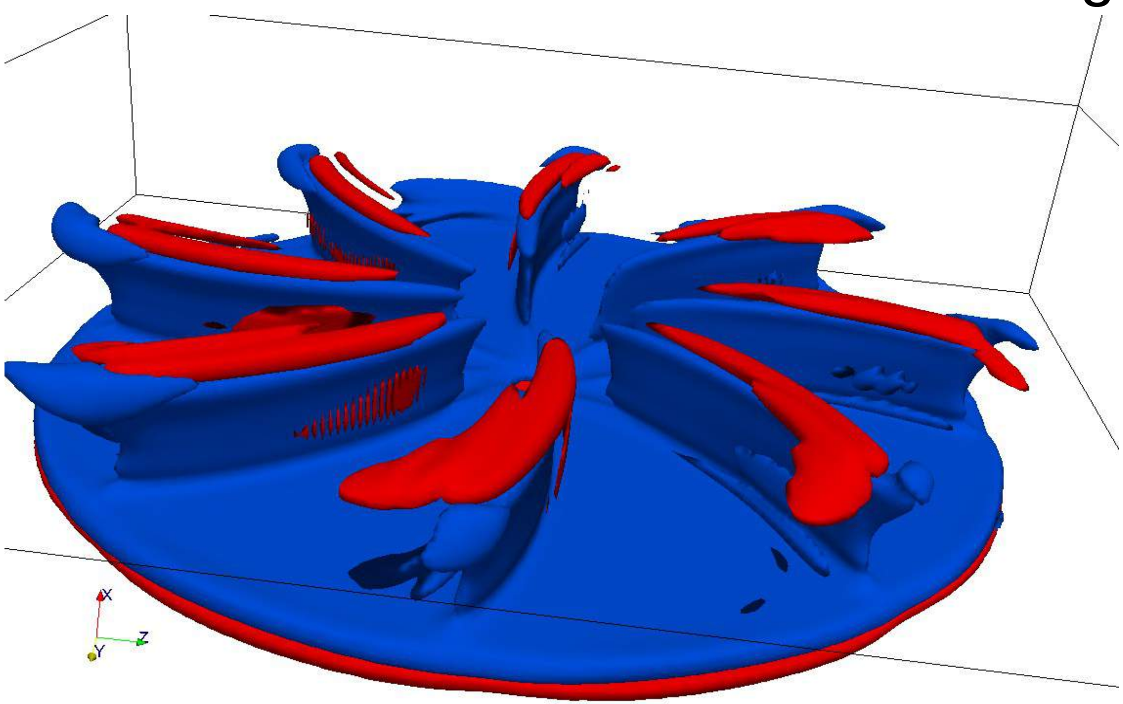
m=0 mode

Topology inside the disk :

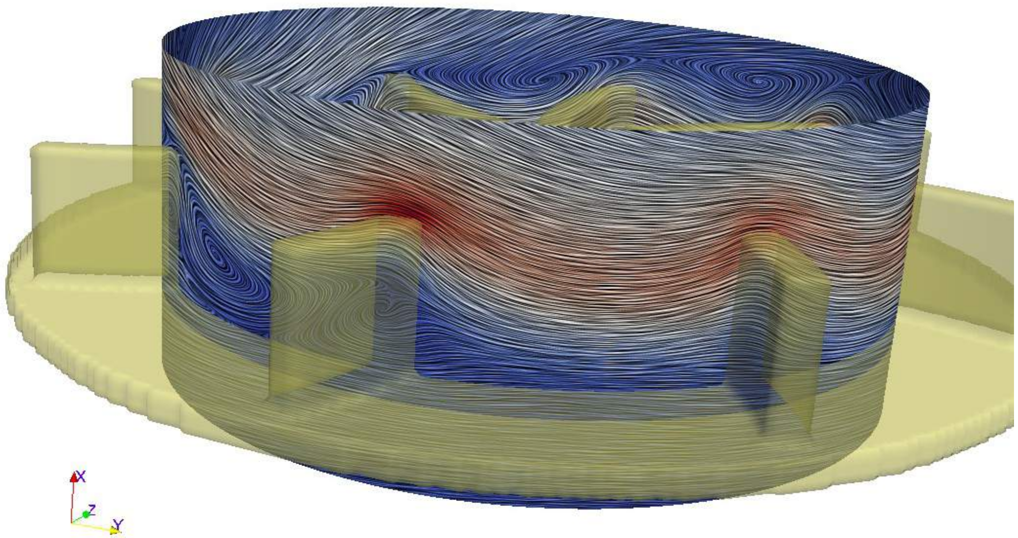


B (magnetic fields)

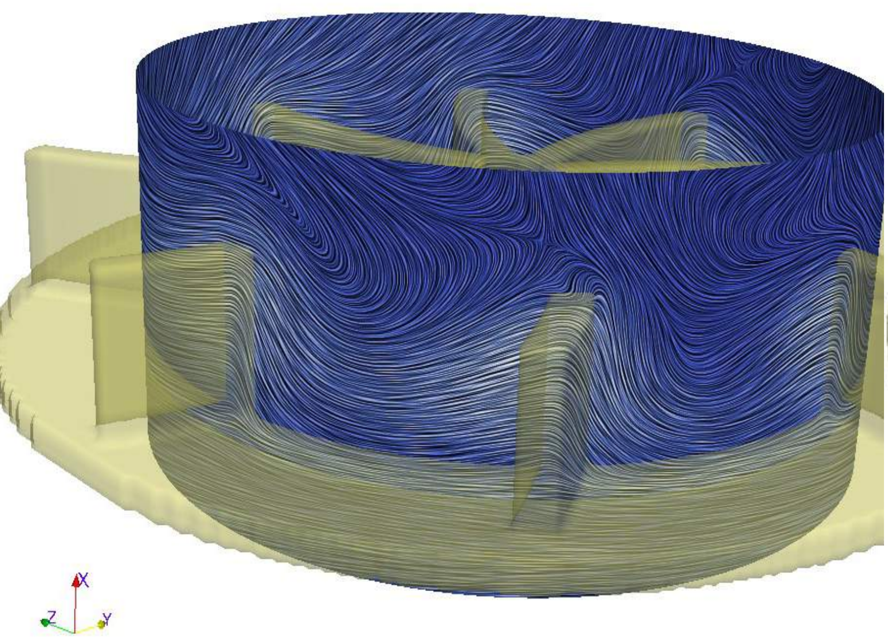
Strong toroidal magnetic field

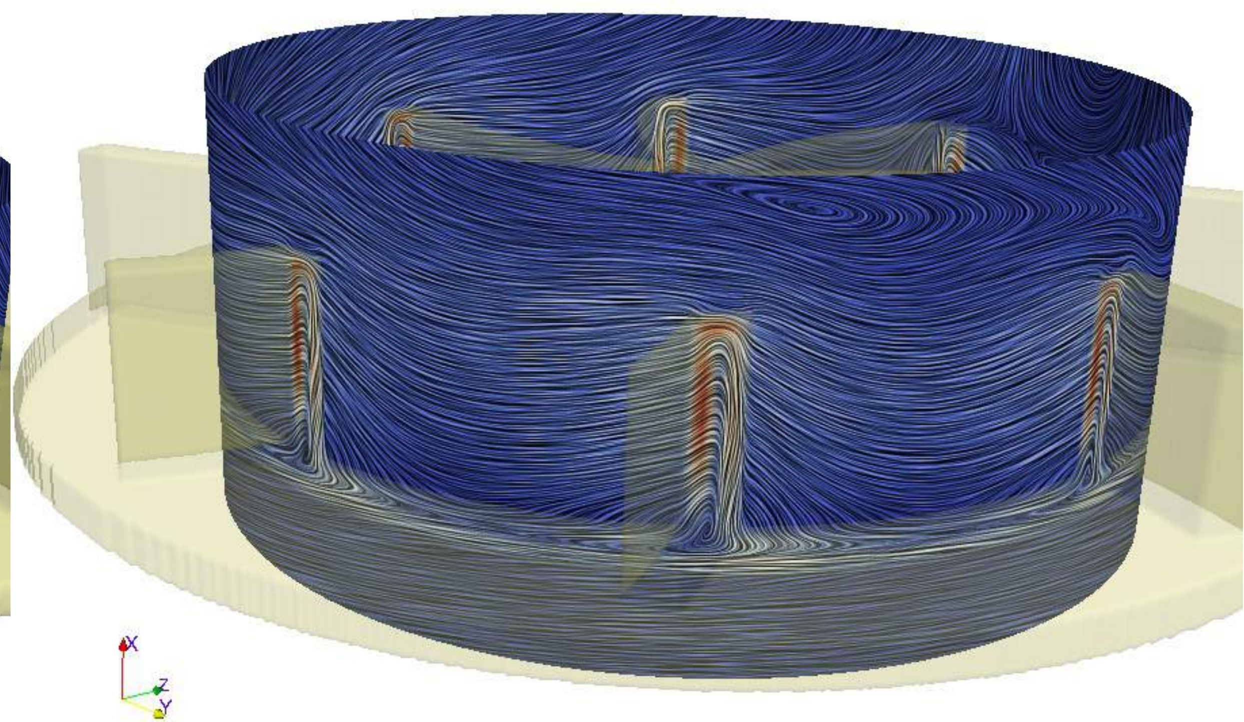


Jr (radial current)



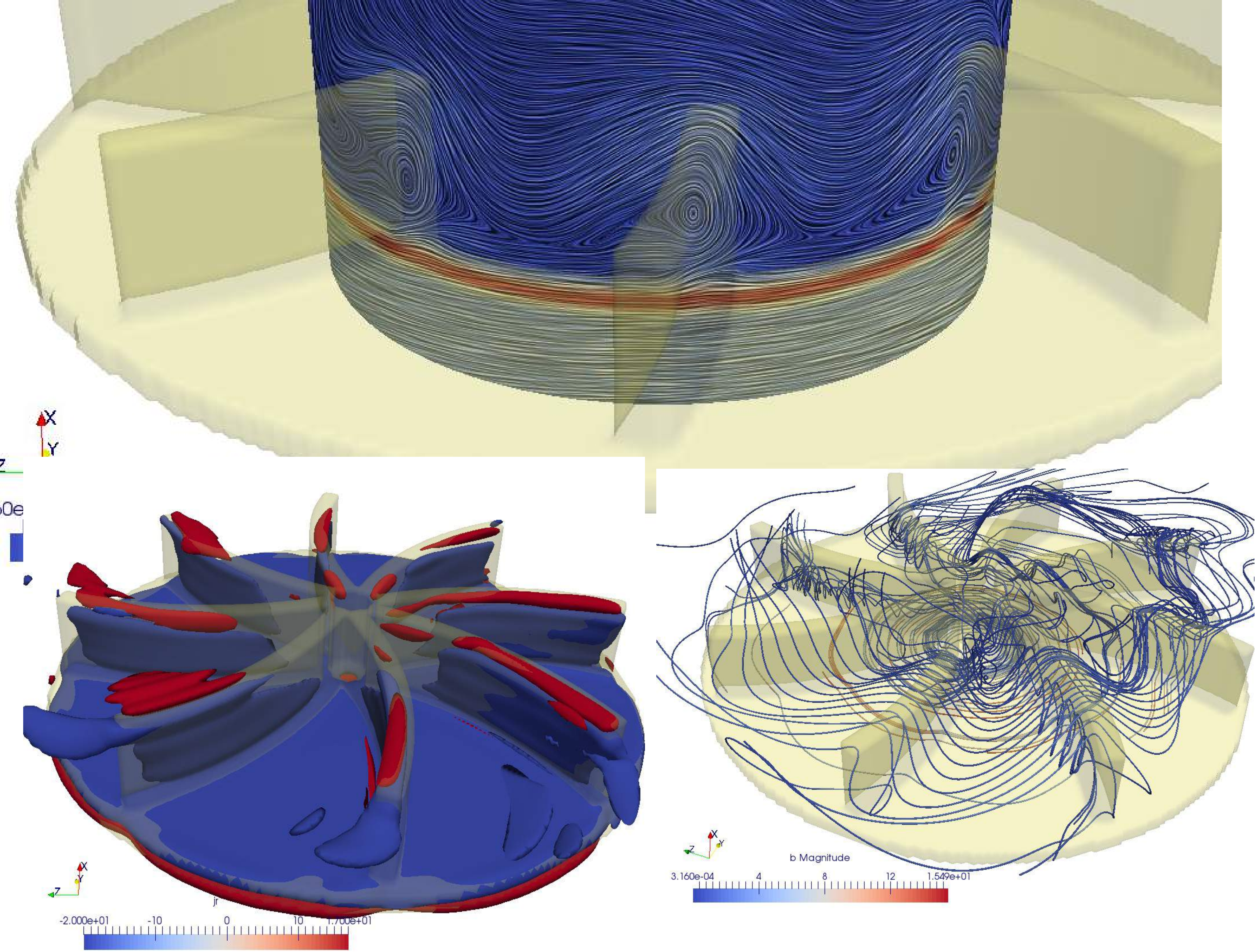
μ =1

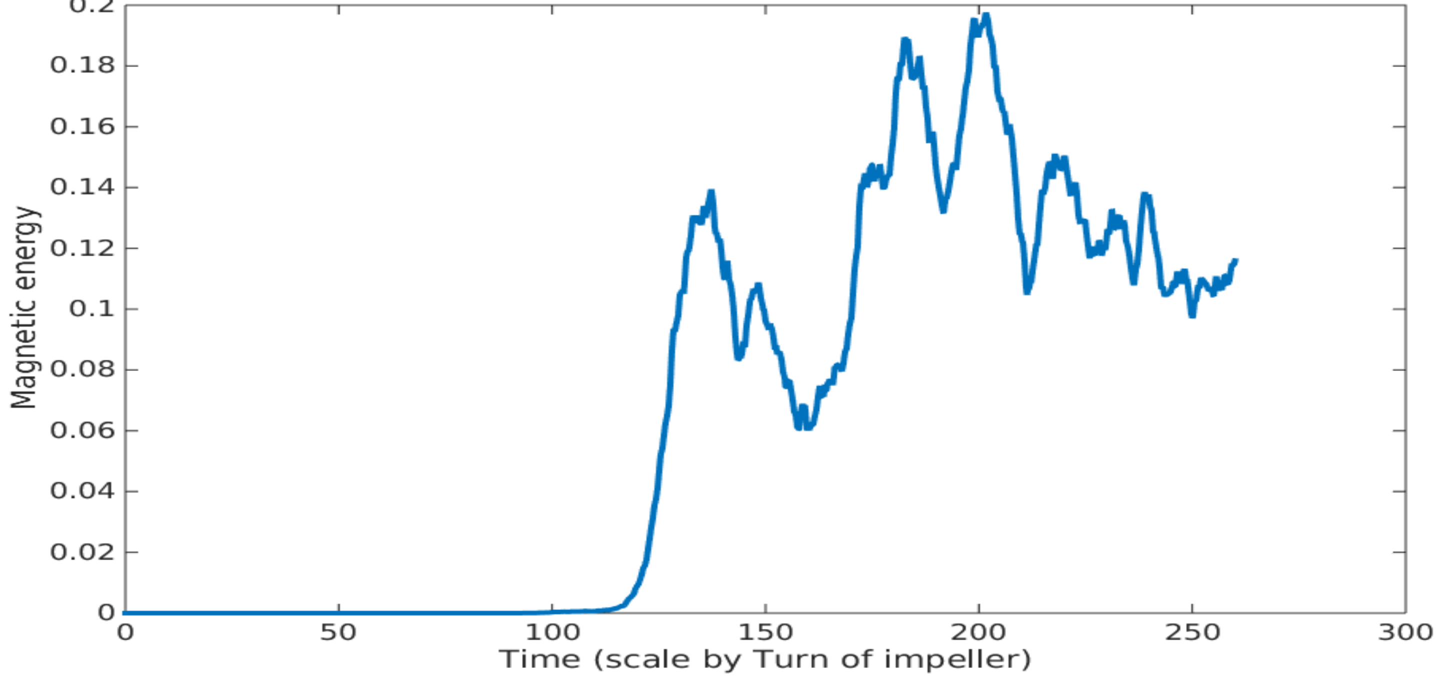




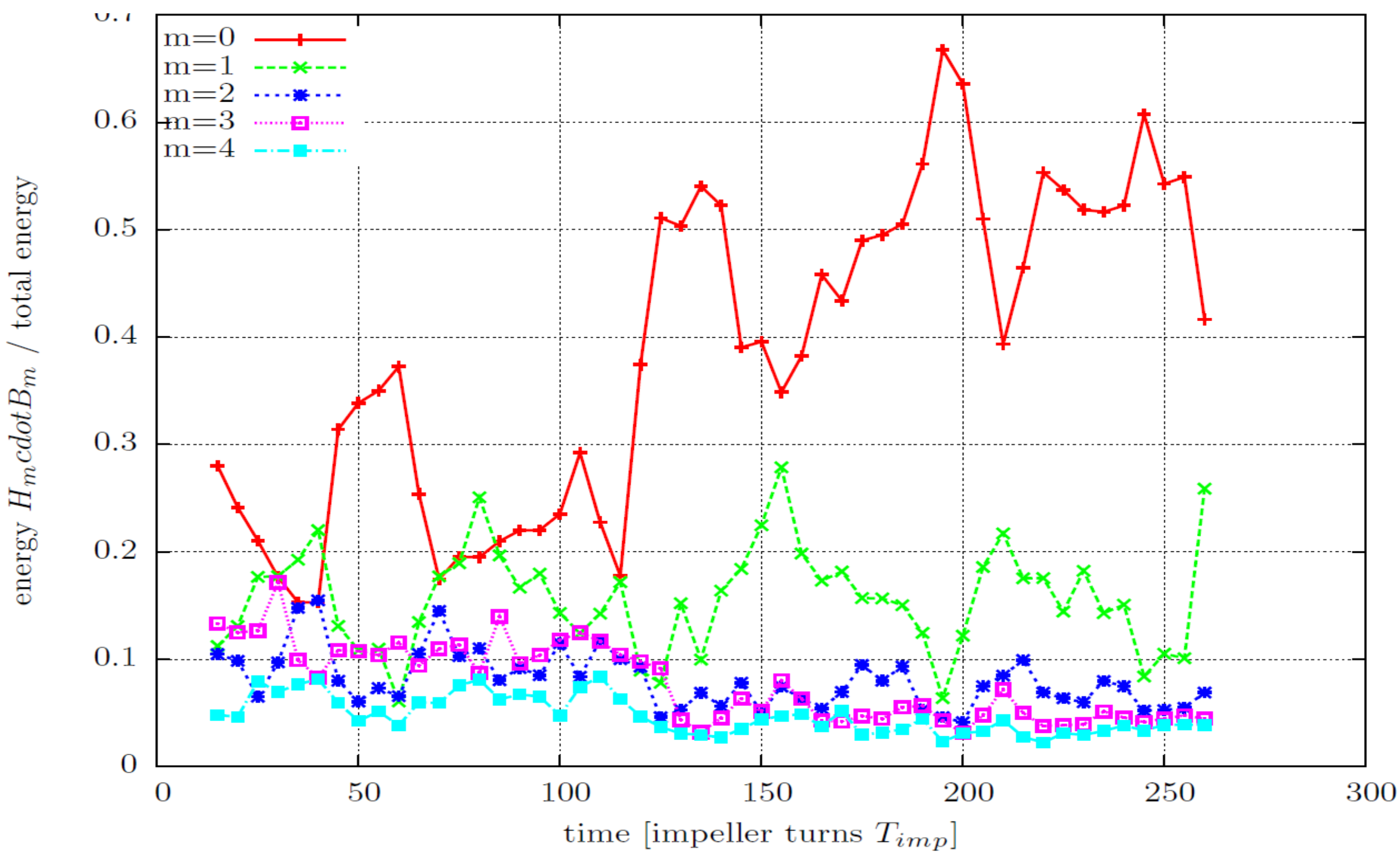
μ =4

μ =8


X
Y
Z
jr
-2.000e+01
-10
0
10
1.700e+01
b Magnitude
3.160e-04
4
8
12
1.549e+01



Mode analysis



Blade effect and winding number

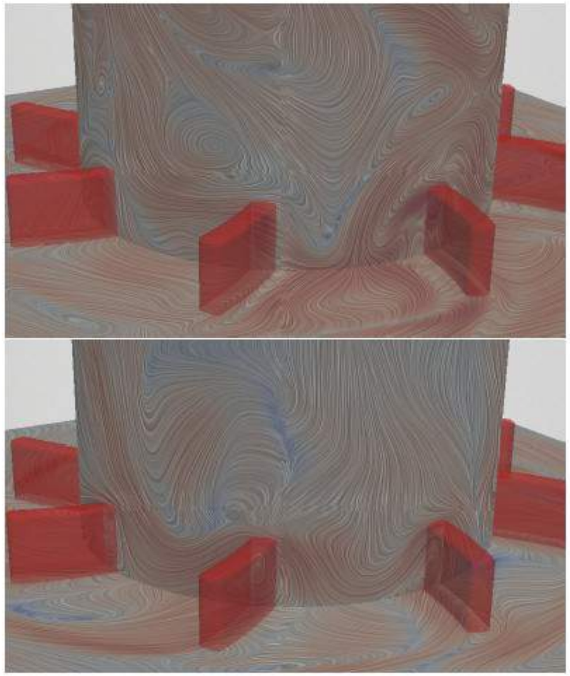
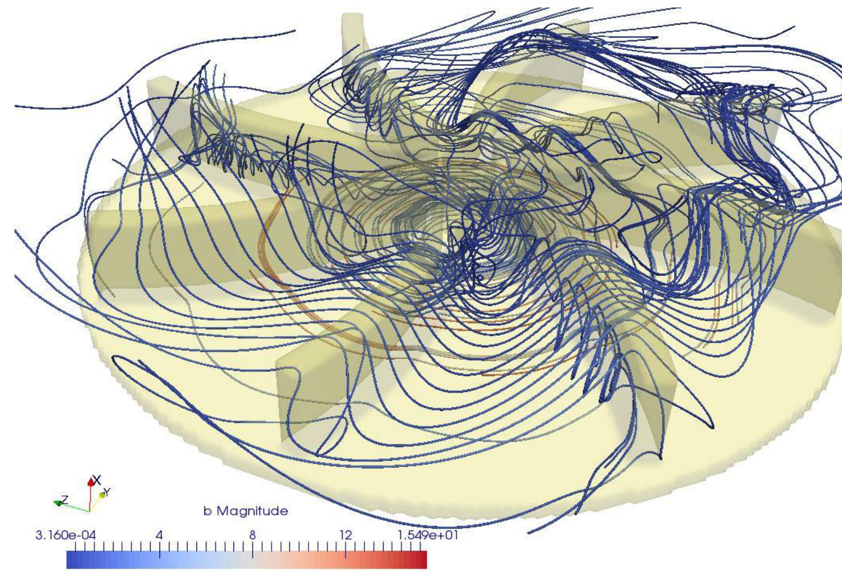
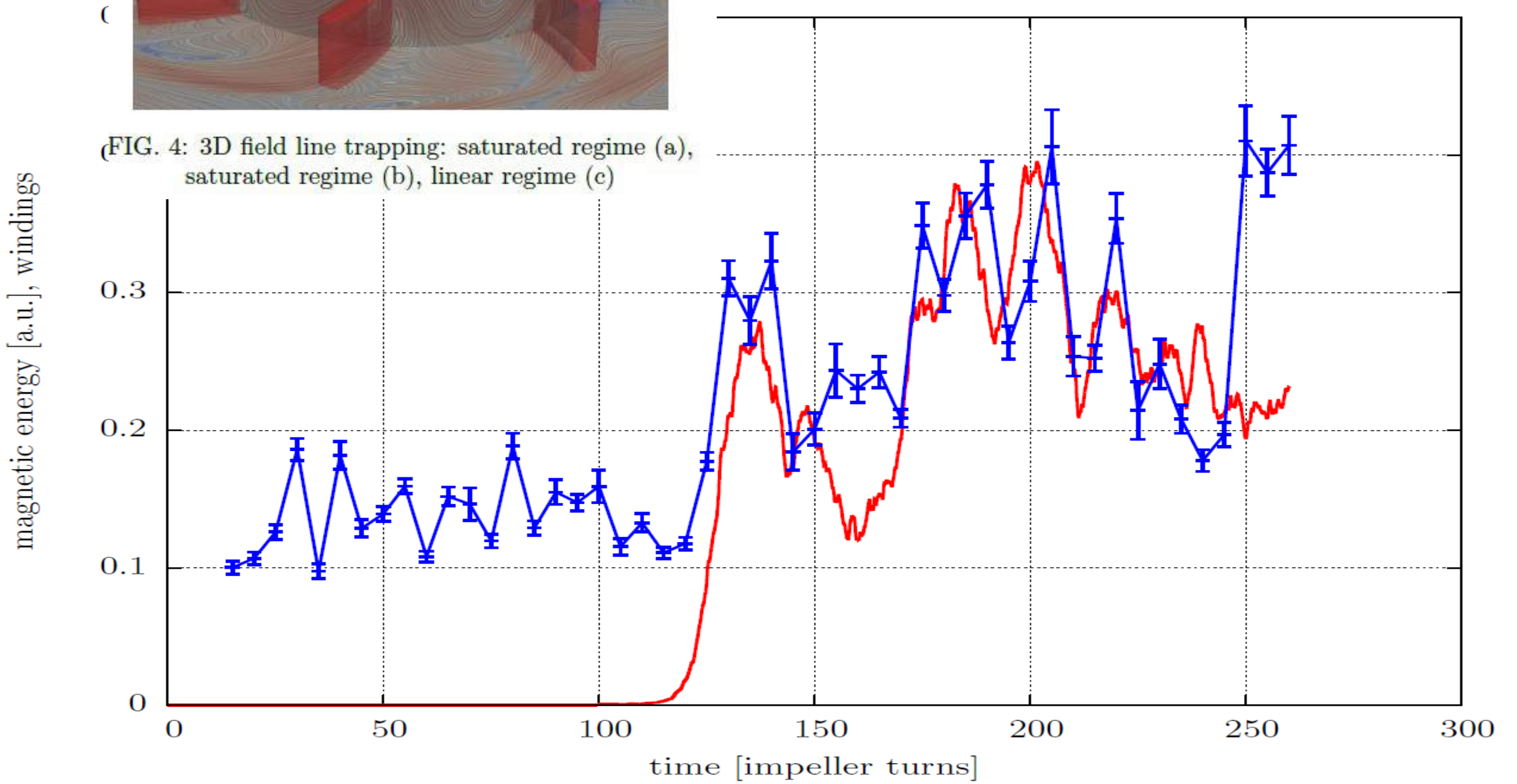
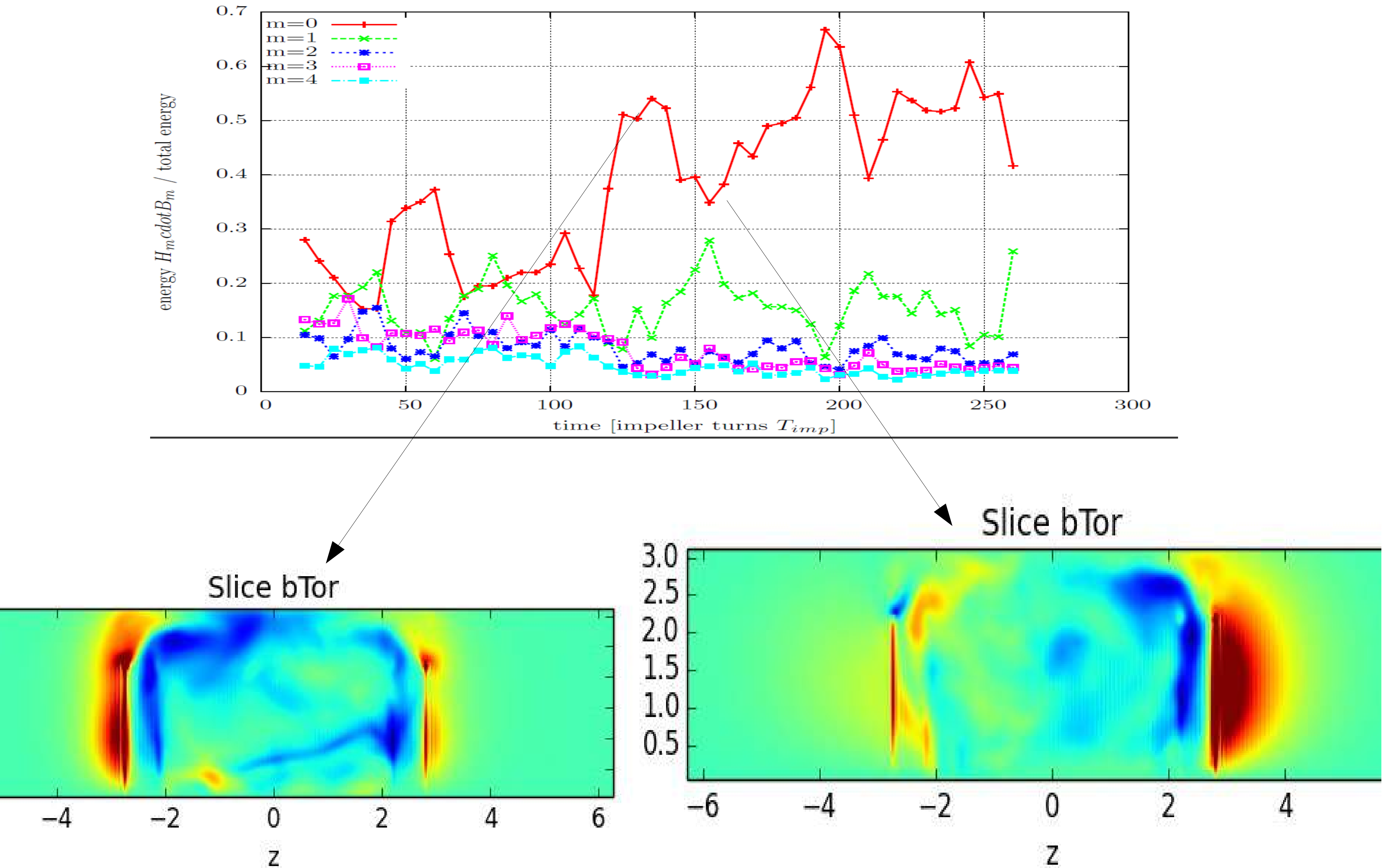


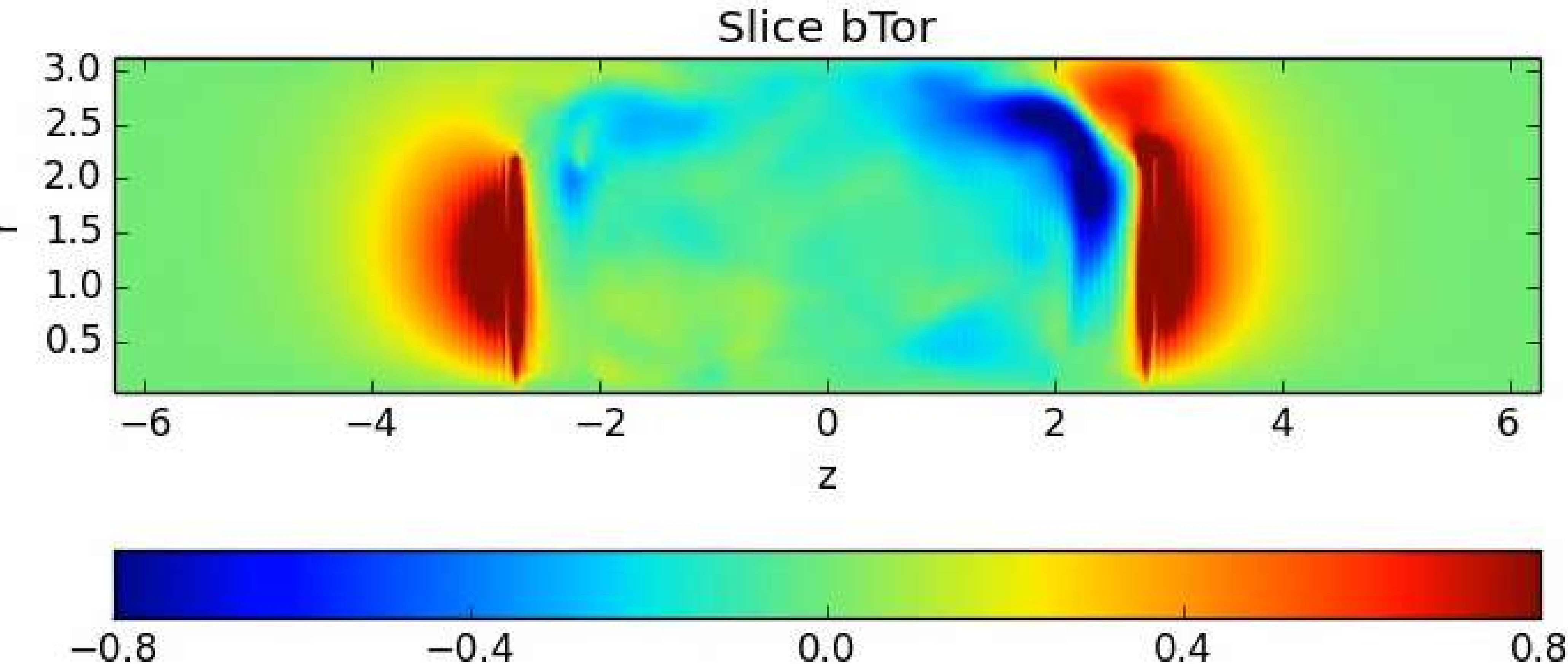
FIG. 4: 3D field line trapping: saturated regime (a), saturated regime (b), linear regime (c)







m=0
m=1
m=2
m=3
m=4
energy $H_m cdot B_m$ / total energy
time [impeller turns T_{imp}]
0
50
100
150
200
250
300
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Slice bTor
Slice bTor
3.0
2.5
2.0
1.5
1.0
0.5
−6
−4
−2
2
4
6
z
z


Slice bTor
3.0
2.5
2.0
1.5
1.0
0.5
r
−6
−4
−2
0
2
4
6
z
−0.8
−0.4
0.0
0.4
0.8

Preliminarily Conclusions

Found similar features of the VKS Dynamo :

- No dynamo → dynamo with m=0 mode
  by increasing magnetic permeability (mu)
- localization of magnetic energy around the impellers.
- Oscillation and reversal of the magnetic fields.
- disk only → no dynamo
- blade only → low magnetic growth rate
- Only one disk dynamo simulation running,
  localization around one disk

Dynamo obtain by increasing the mu is not only a boundary effect,
But a geometrical one :

Crucial role of the blade (with hight permeability)

↓

Magnetic fluid-structure Dynamo