Few things we learned from the Von Karman Sodium dynamo

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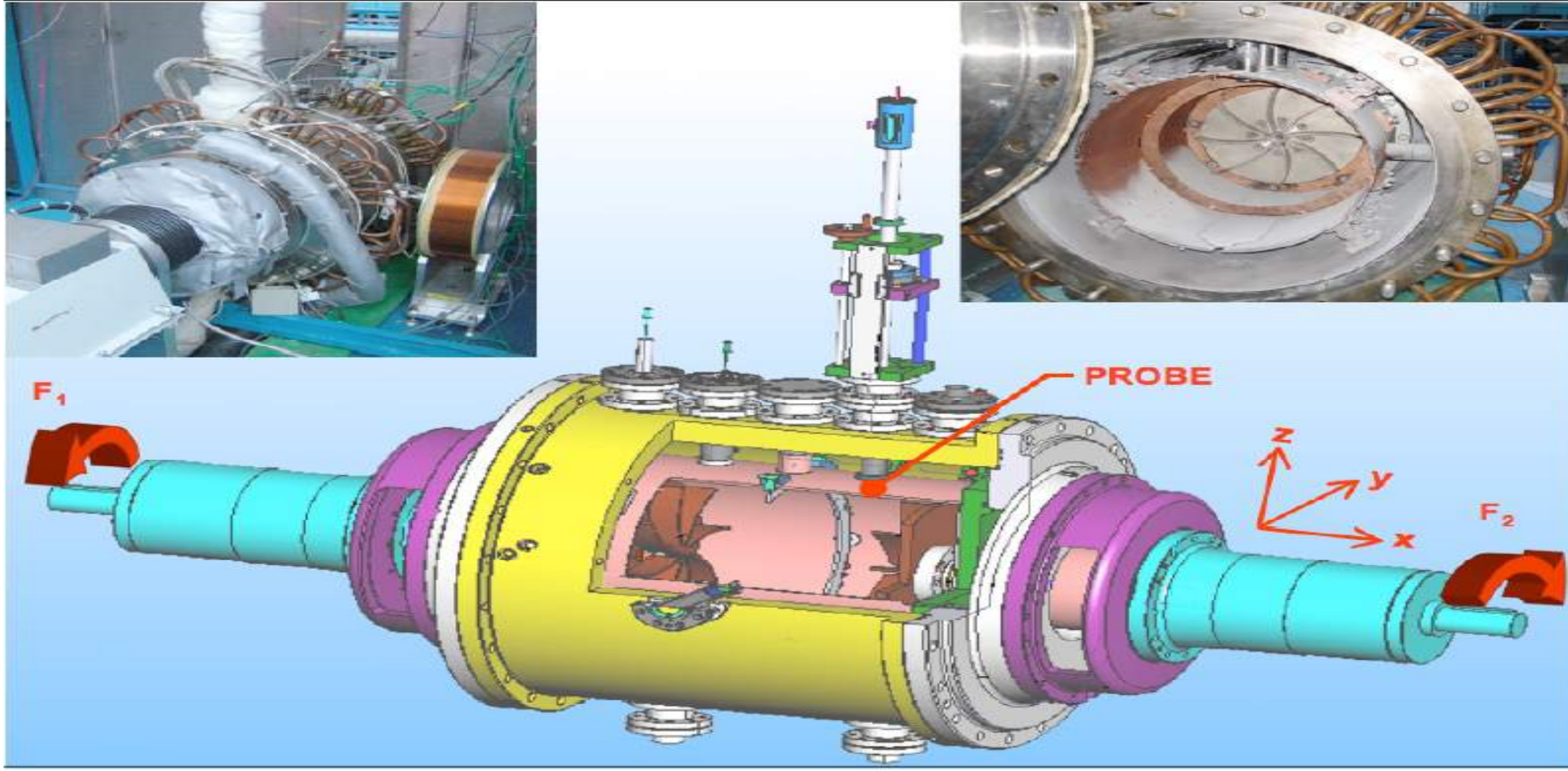
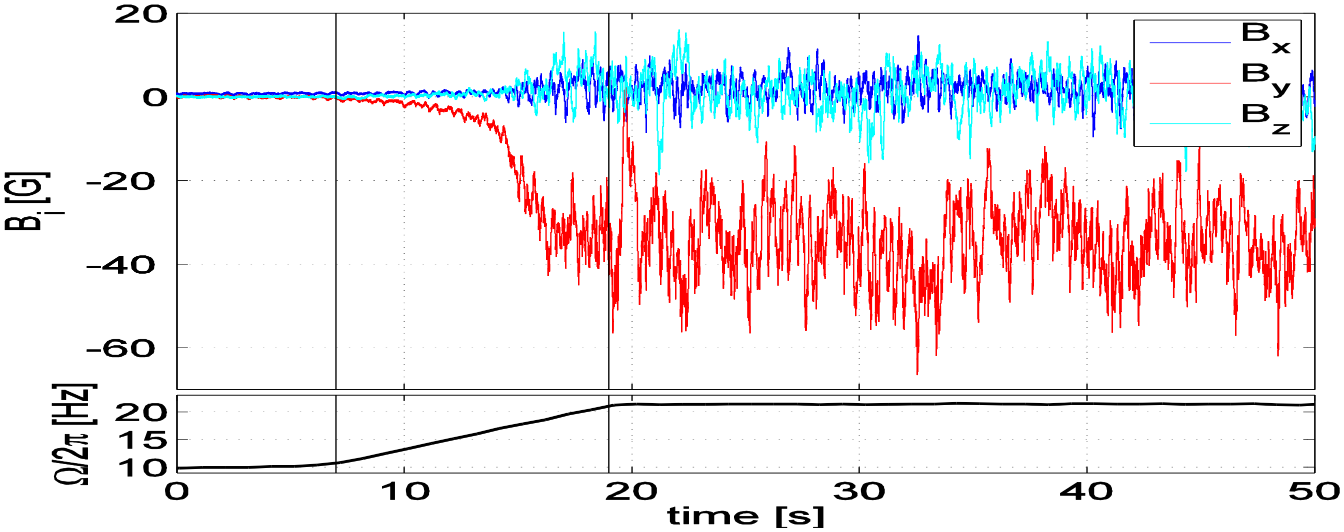


2 x 150 kW motors

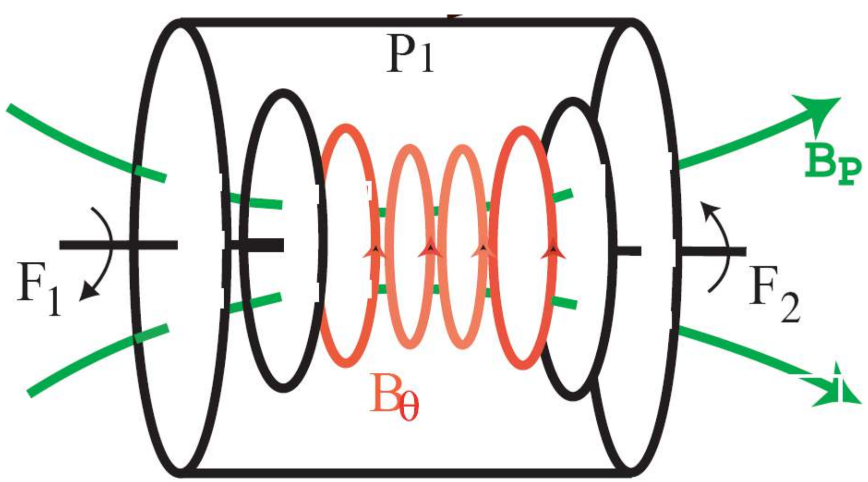
150 liters Liquid Na
(100-160 C)

Soft Iron propellers



Structure of the time-averaged field (exact counter rotation)

An axial dipole



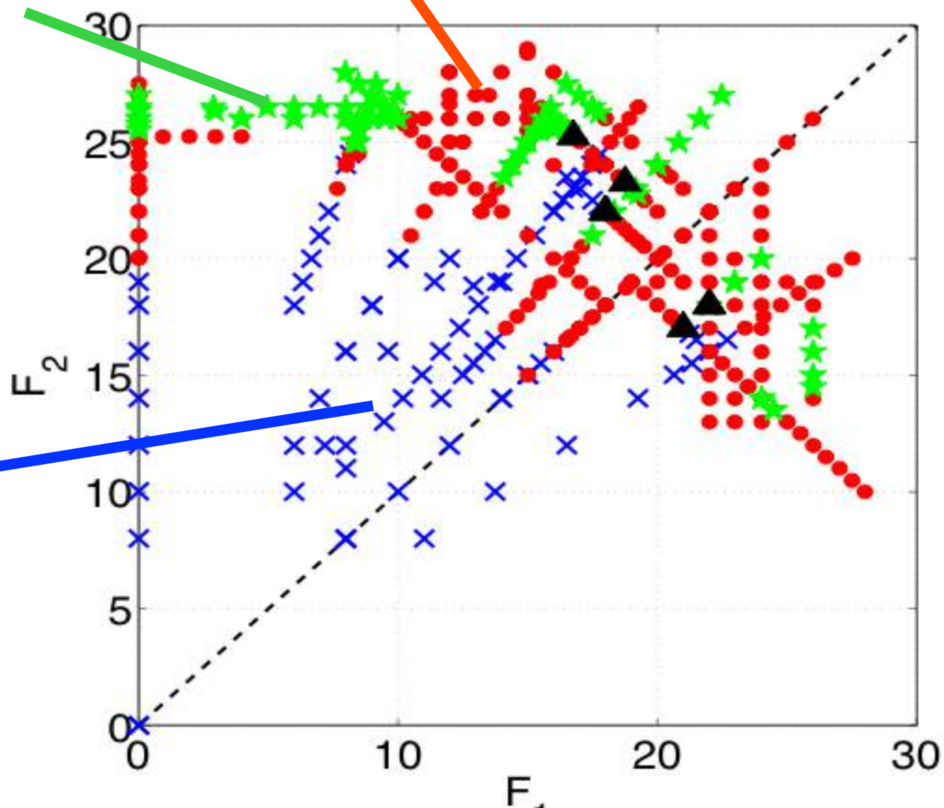
No reversals in exact counter rotation (stationary regime)

Parameter space
(disks rotate at different speeds)

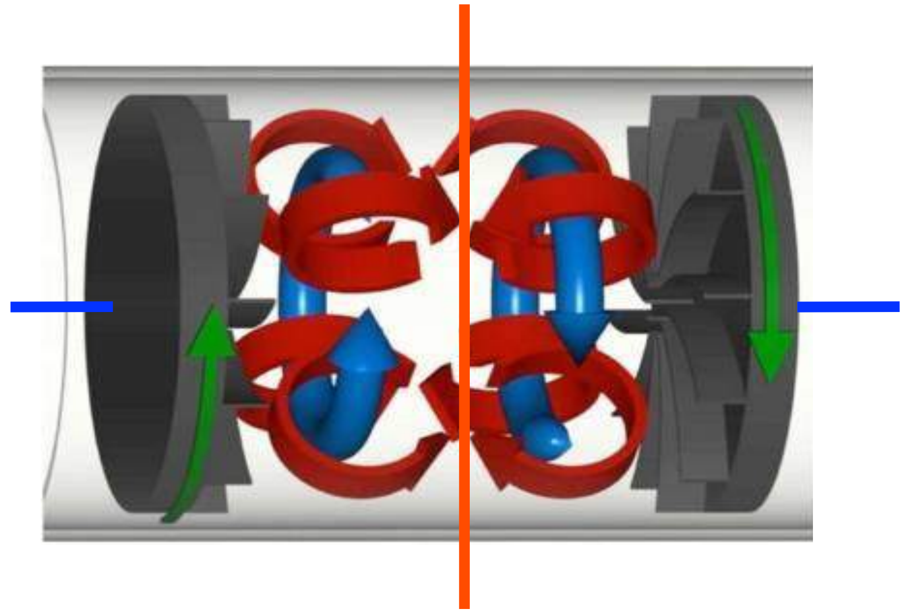
Stationary regimes

Dynamical regimes (such as random or periodic reversals)

No dynamo



No dynamical regime in exact counter-rotation



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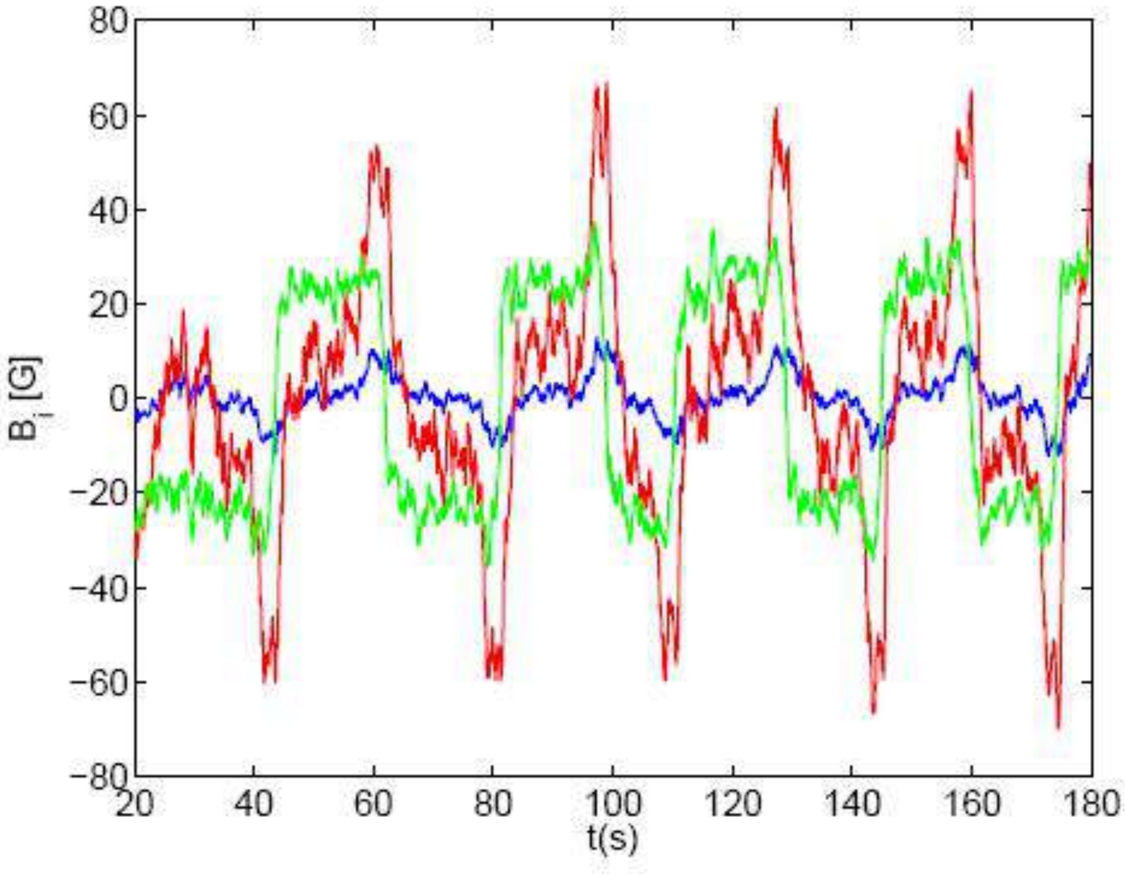
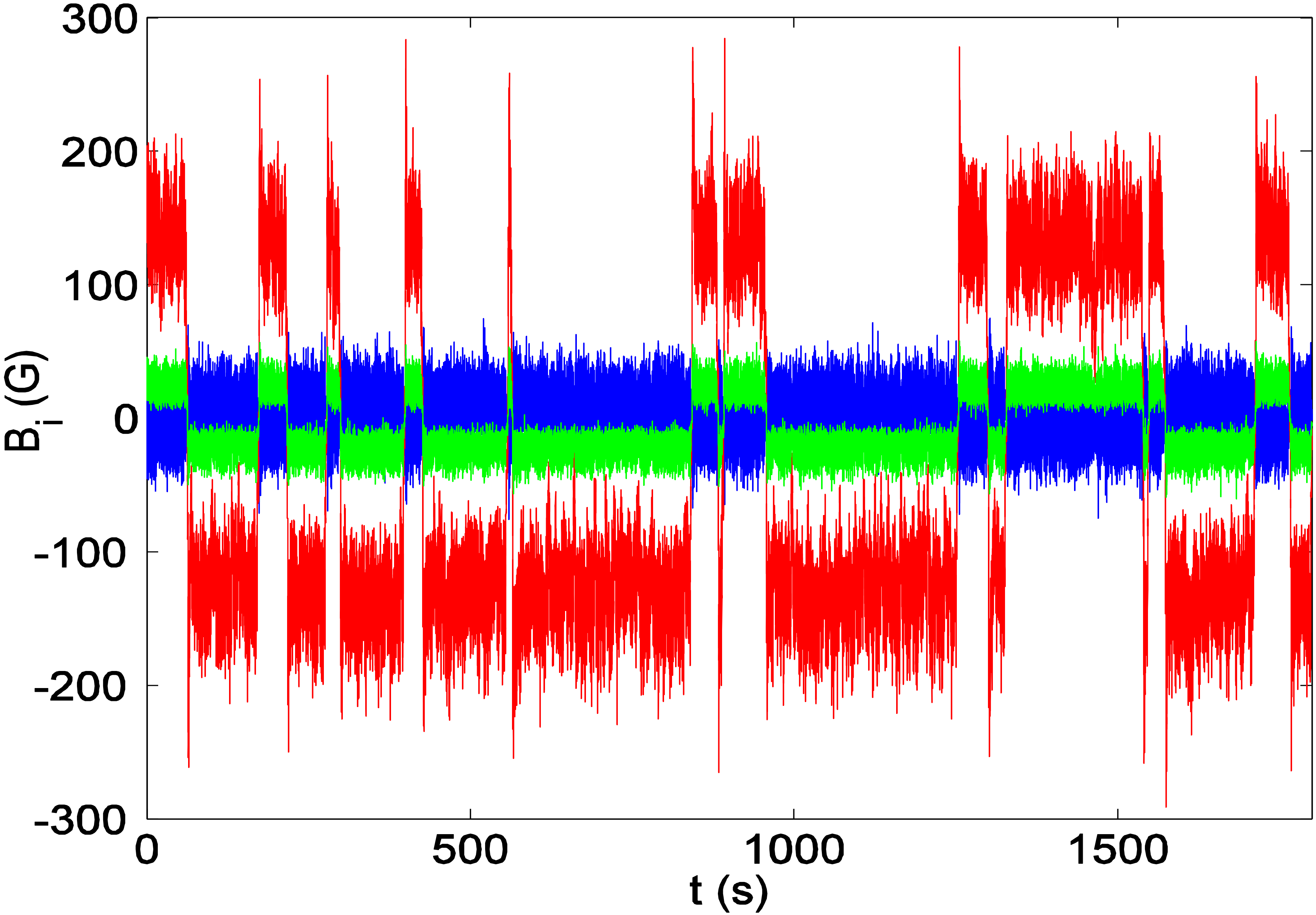
Nonlinear oscillations



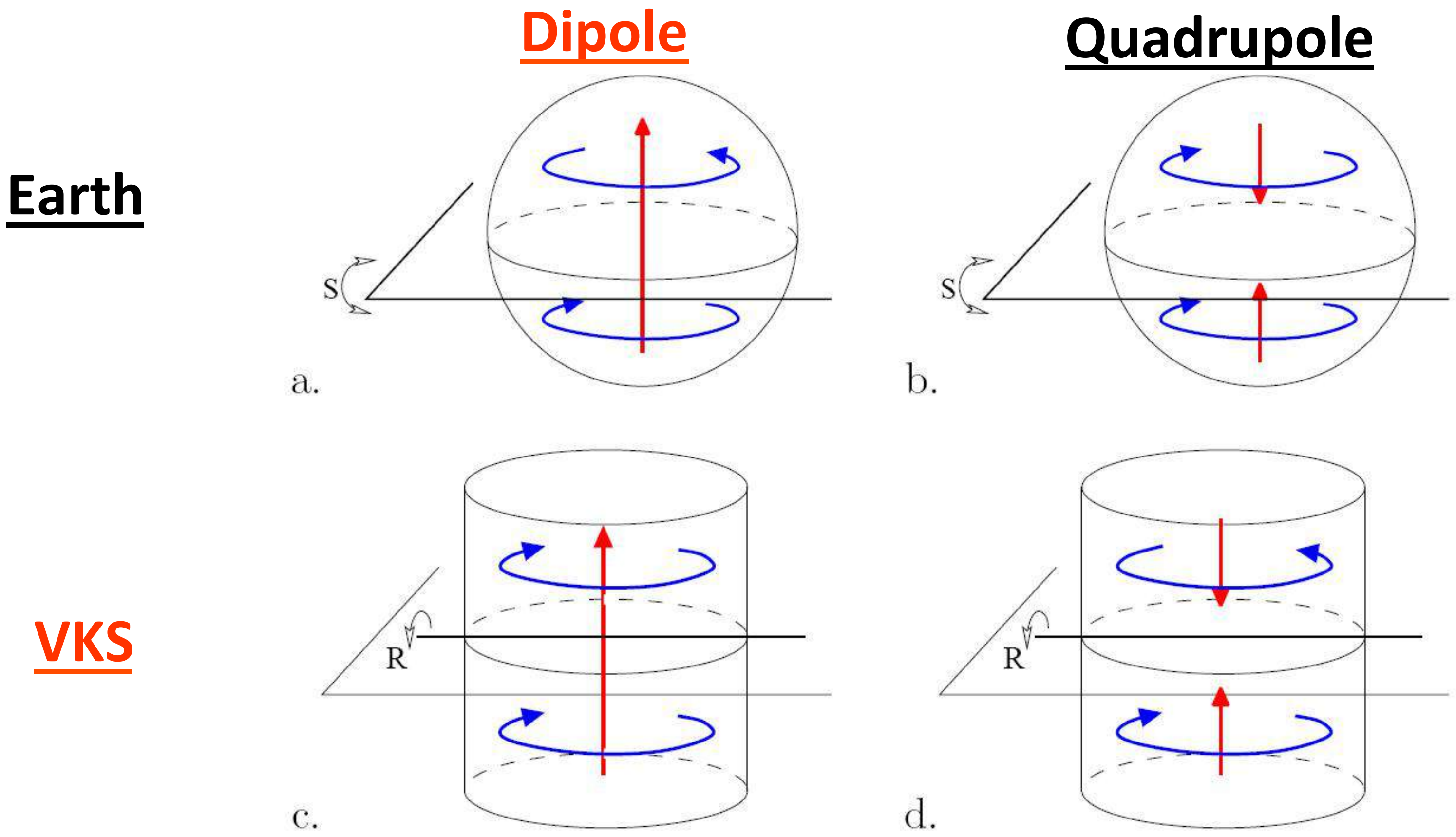
Reversals sharing lots of similarities with the geodynamo



Low dimensional model of field dynamics

with S. Fauve, E. Dormy (LRA) and J.-P. Valet (IPGP)

Symmetry properties of the modes



Equation for dipole and quadrupole

$$\mathbf{B}(r,t) = d(t)\mathbf{D}(r) + q(t)\mathbf{Q}(r)$$

Let A=d+i q, $\dot{A} = \mu A + \nu \bar{A} + \beta_1 A^3 + \beta_2 A^2 \bar{A} + \beta_3 A \bar{A}^2 + \beta_4 \bar{A}^3$

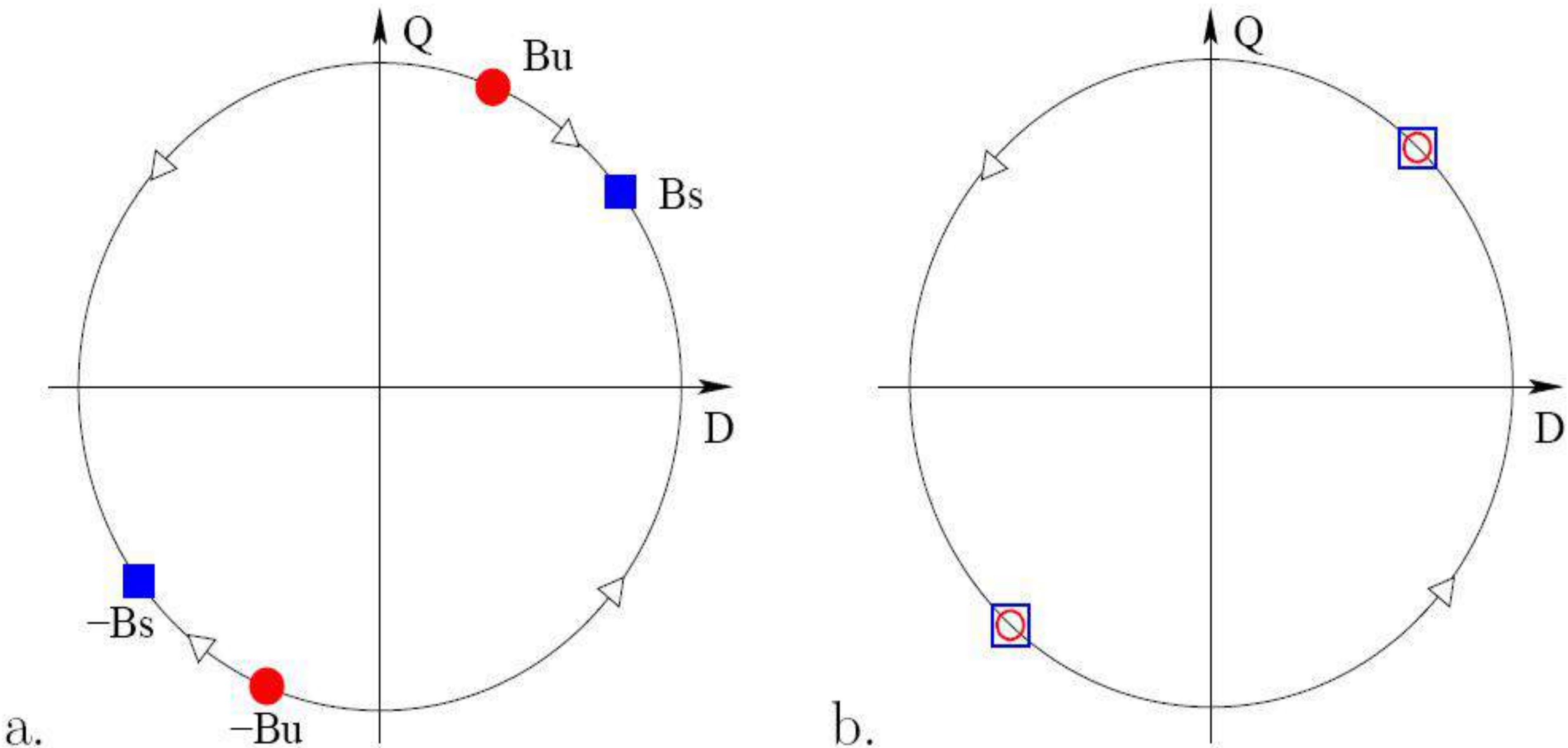
Phase equation $A = r \exp(i\,\theta)$

Simplified form $\dot{\theta} = \mu_i + \nu_i \cos(2\theta) - \nu_r \sin(2\theta)$

If symmetric flow, Unchanged under $d \to -d$, $q \to q$ i.e. $A \to -\bar{A}$ so $\mu_i = 0$

Magnetic field

$$\dot{\theta} = \mu_i - \nu_r \sin(2\theta)$$



Magnetic field

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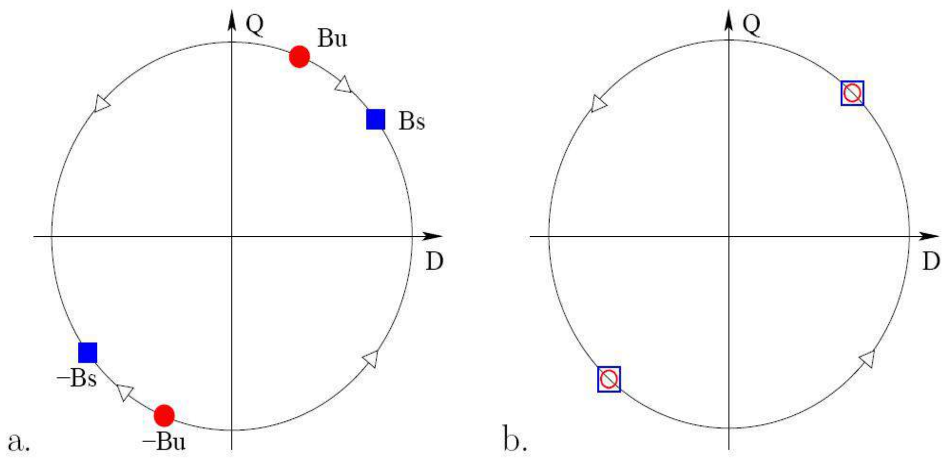
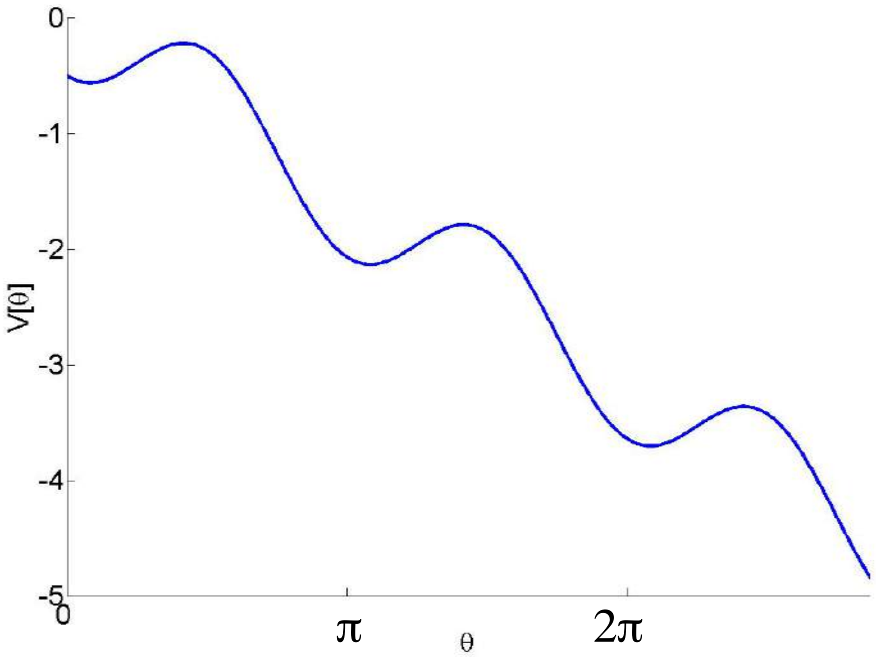
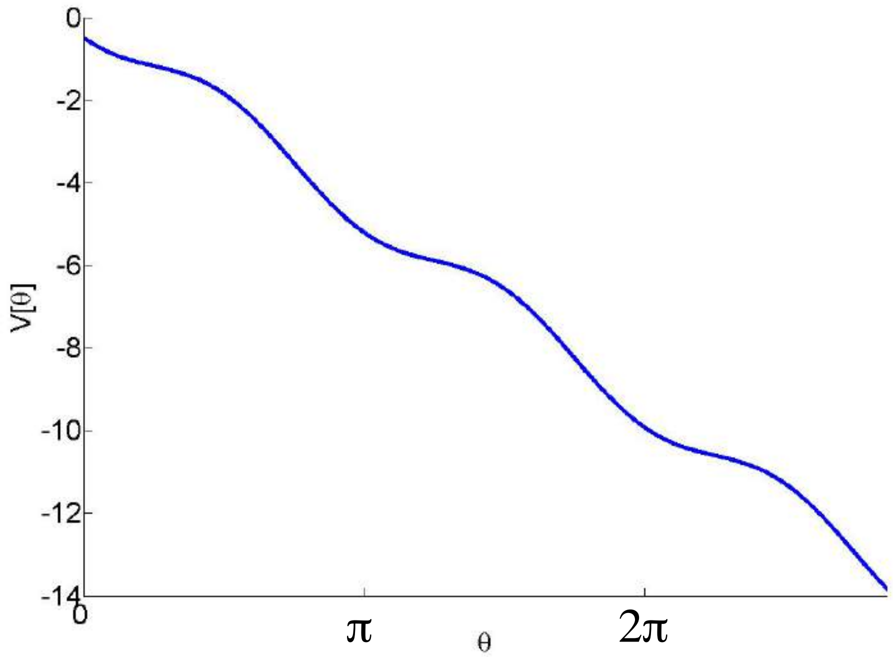


Evolution in a potential

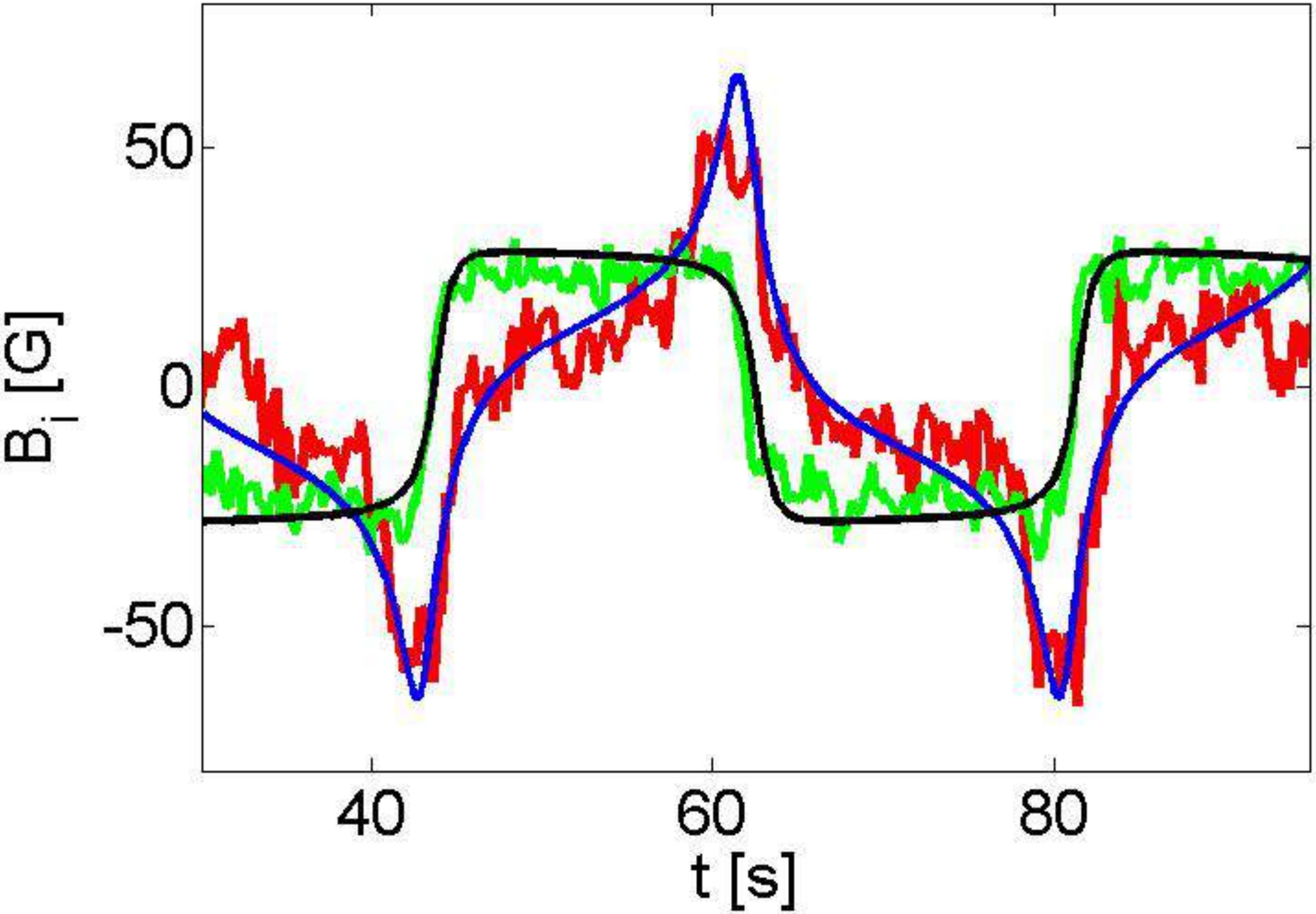
$$\dot{\theta} = -\frac{\partial V}{\partial \theta}$$

$$V[\theta] = -\mu_i\theta - \nu_r \cos(2\theta)/2$$

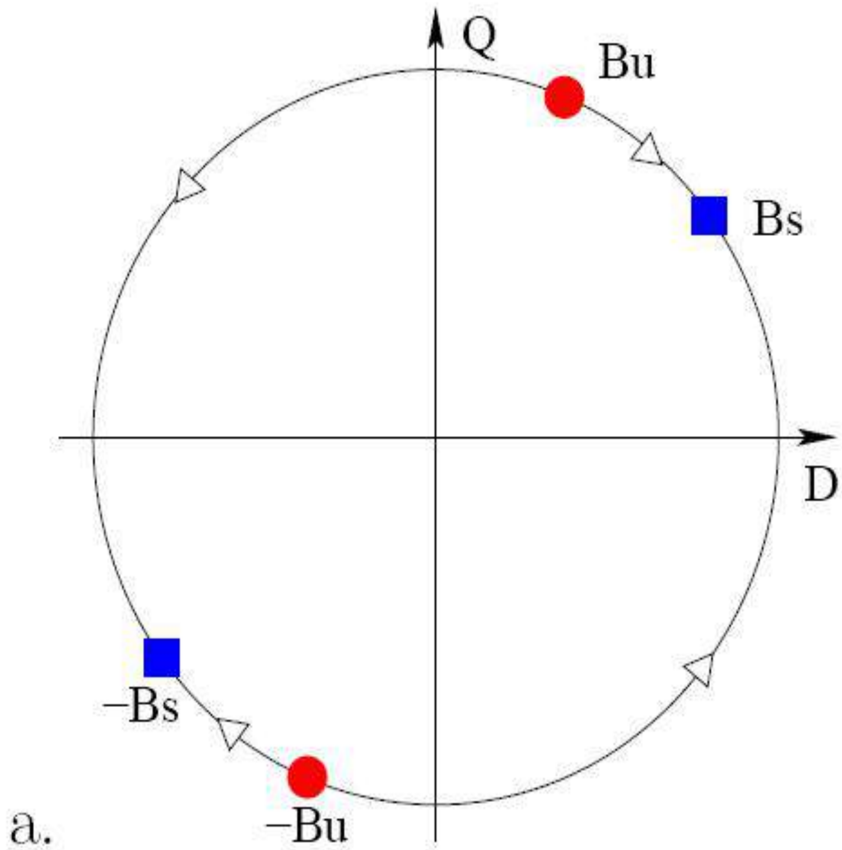


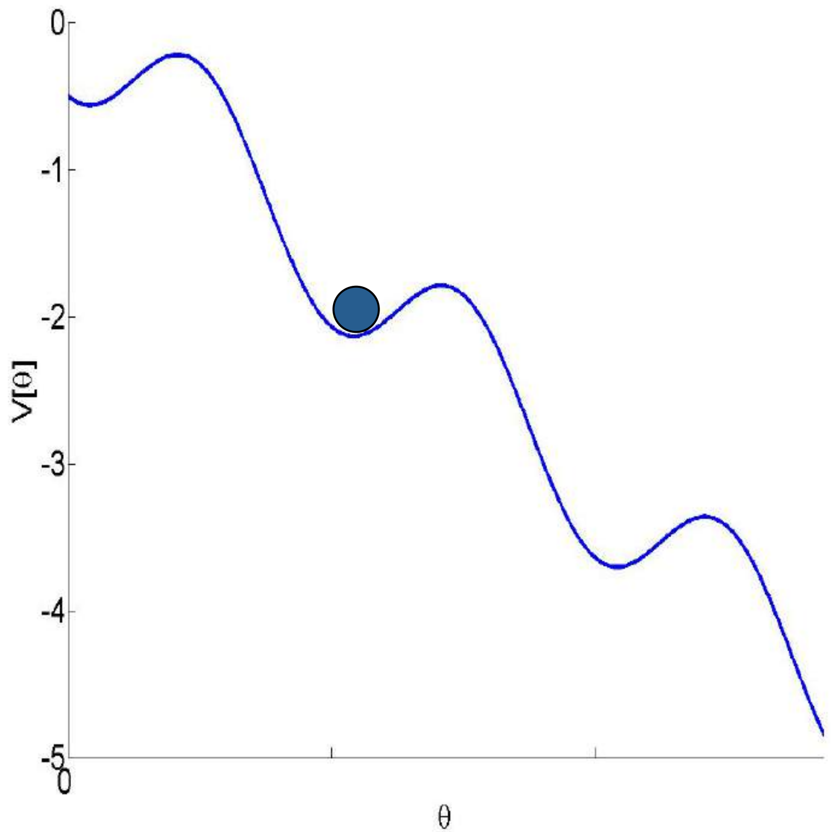


Comparison between time series



Effect of turbulent fluctuations: reversals





Predictions

Origin and shape of reversals :

- Two modes close to a saddle-node bifurcation
- Slow phase followed by a fast phase

Origin and shape of excursions:

- Aborted reversals
- Same initial phase as reversals but end up without overshoot

This is observed in the VKS reversal time series and in the geodynamo one.

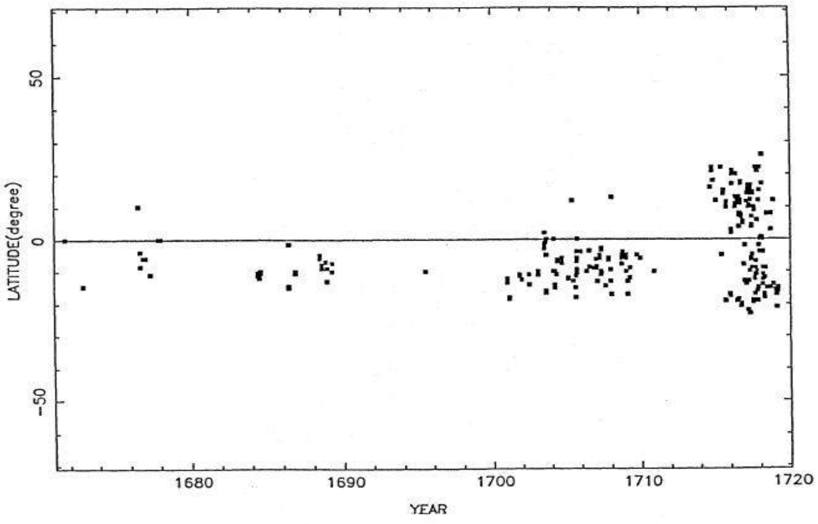
Some astrophysical dynamos are hemispherical

The Sun during

Maunder Minimum

Few sunspots,

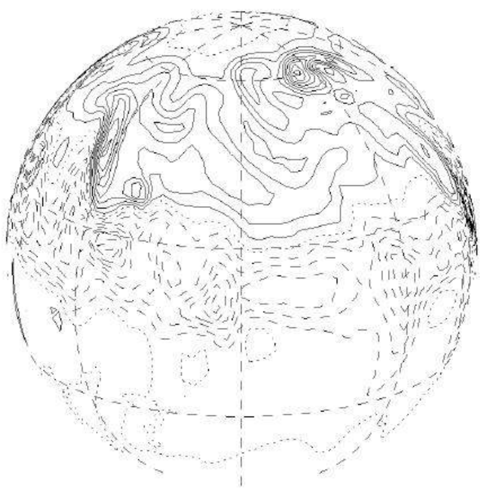
all in the Southern hemisphere



Ribes and Nesme-Ribes A&A 93

* Mars Surface (Stanley et al. Science 2008)

* Numerical Simulations

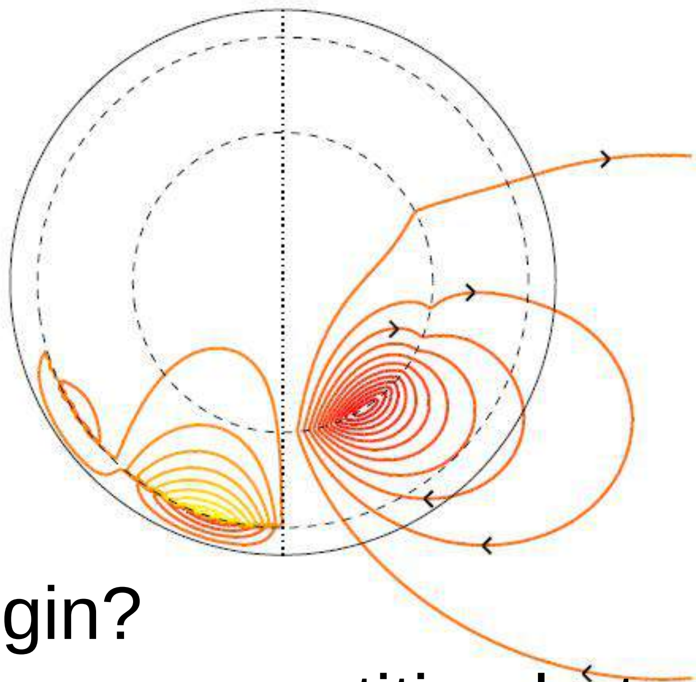


Grote et Busse PRE 2001

Landau et Aubert 2010

An analytical model (B. Gallet Ph.D. , PRE 2009)

- Parametrisation of the induction effect
- Broken symmetry causes reversals



95% of the energy in the Southern hemisphere (at r=R)

Symmetry breaking terms smaller than 1% of the symmetric ones

Origin?

Also a competition between dipole and quadrupole !

If d+i q=r exp(i θ), then

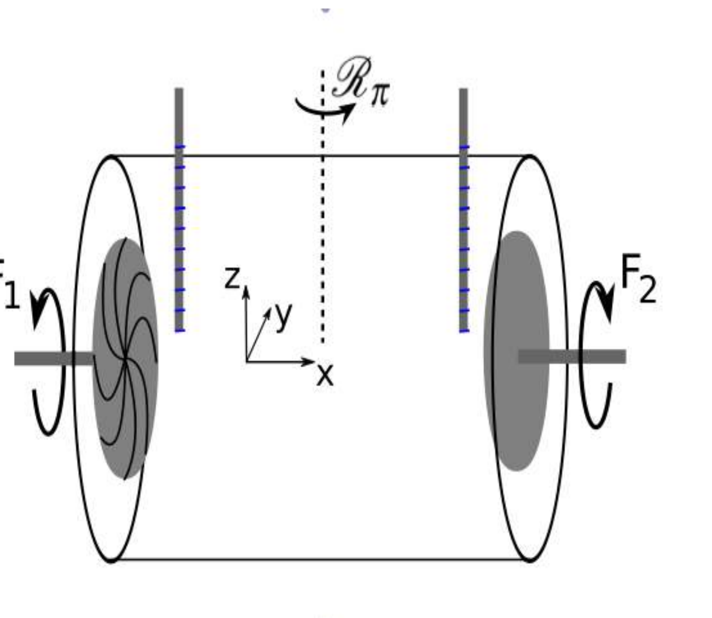
$$\dot{\theta} = \mu_i + \nu_i \cos(2\theta) - \nu_r \sin(2\theta)$$

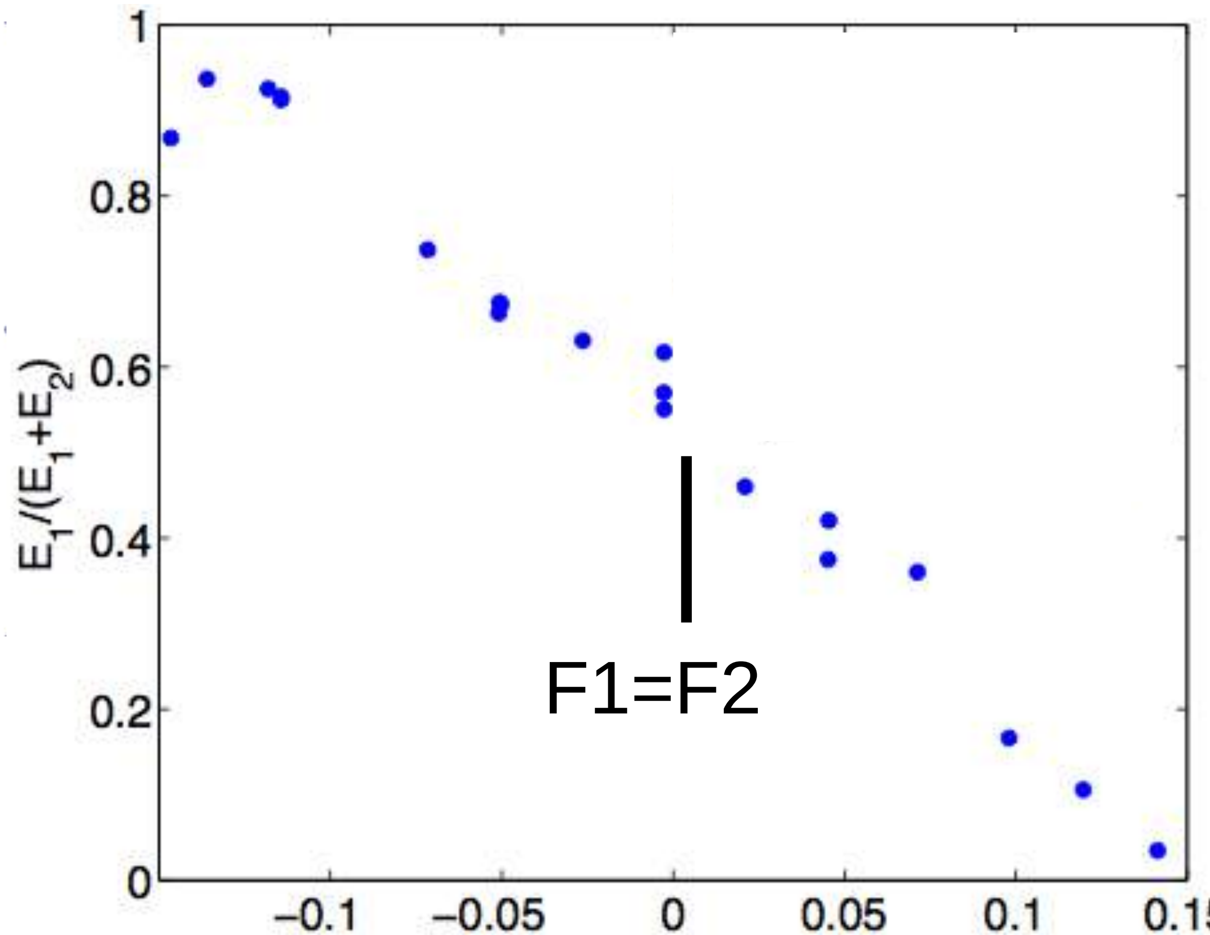
If ν_i arge, the field is D(r)+Q(r) (or D(r)-Q(r)):

These are hemispherical dynamos!

Localized dynamo (B. Gallet et al. PRL 2012)

E1 is magnetic energy close to disk 1





(F1-F2)/(F1+F2)

First experimental observation of localized dynamo

Are the physical properties of the Earth liquid core or the Sun convective zone or the galaxy uniform ?

How large are the variations of electrical conductivity?

Is there an effect on the dynamo of the variations of electrical conductivity in the bulk of the fluid?
(work with A. Alexakis, C. Gissinger and S. Fauve)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times \left(\frac{1}{\sigma} \nabla \times (\frac{\mathbf{B}}{\mu_0}) \right)$$

$$\eta = (\sigma \mu_0)^{-1}$$

$$\eta = \eta_0 + \delta\eta$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\delta\eta \nabla \times \mathbf{B}) + \eta_0 \nabla^2 \mathbf{B} \,.$$

Calculation using scale separation (first order smoothing)

Assume v and eta varies on a small lengthscale l

$$\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$$

Large scale field

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{v} \times \mathbf{b} \rangle - \langle \delta\eta \nabla \times \mathbf{b} \rangle) + \eta_0 \nabla^2 \langle \mathbf{B} \rangle$$

Small scale field

$$-\eta_0 \nabla^2 \mathbf{b} = \langle \mathbf{B} \rangle . \nabla \mathbf{v}$$

The first term is the usual alpha effect $\langle vxb \rangle = \alpha \langle B \rangle$

$$\alpha^h_{u,j} = (2\pi)^{-3} i \Sigma_k \frac{\mathbf{k}_j}{\eta_0 \mathbf{k}^2} \left(\hat{\mathbf{v}}(-\mathbf{k}) \times \hat{\mathbf{v}}(\mathbf{k}) \right)_u$$

The second term is new

$$\alpha^{\sigma}_{u,j}\mathbf{B}_j = -\langle \delta\eta \nabla \times \mathbf{b} \rangle = (2\pi)^{-3}\Sigma_k \frac{\mathbf{k}.\langle \mathbf{B} \rangle}{\eta_0 \mathbf{k}^2} \hat{\delta\eta}(-\mathbf{k}) \left(\mathbf{k} \times \hat{\mathbf{v}}(\mathbf{k})\right)_u$$

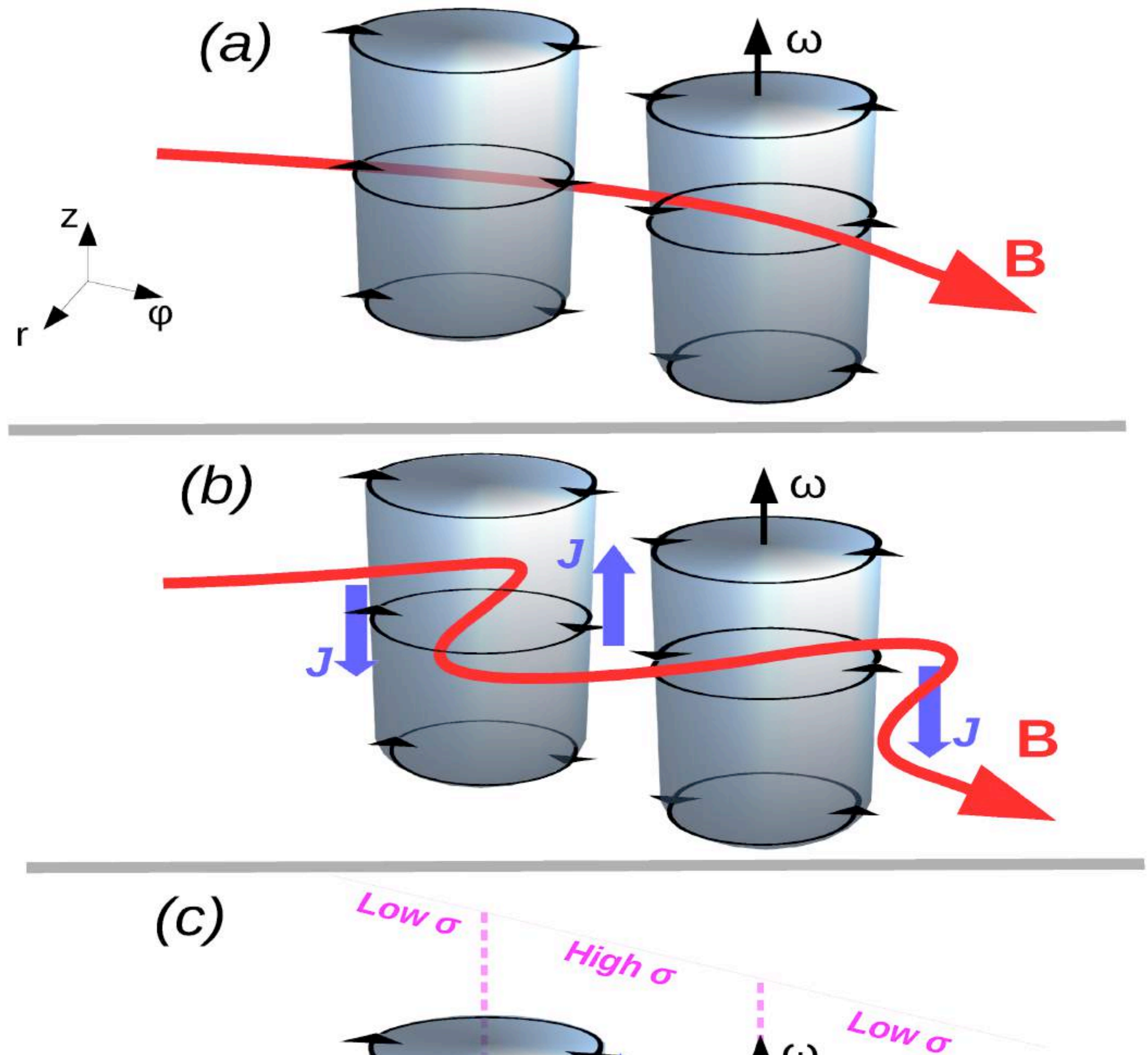
Introducing the vorticity $\mathbf{\Omega} = \nabla \times \mathbf{v}$

$$\alpha^{\sigma}_{u,j} = -(2\pi)^{-3} i \Sigma_k \frac{\mathbf{k}_j}{\eta_0 \mathbf{k}^2} \left(\hat{\delta\eta}(-\mathbf{k}) \hat{\mathbf{\Omega}}_u(\mathbf{k}) \right) = -(2\pi)^{-3}\Sigma_k \frac{\hat{\delta\eta}(-\mathbf{k}) \partial_j \hat{\mathbf{\Omega}}_u(\mathbf{k})}{\eta_0 \mathbf{k}^2}$$

Effect requires correlation between diffusivity variations and gradients of vorticity

Creates a current in the direction of vorticity

No need for helicity


(a)
ω
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r
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J
J
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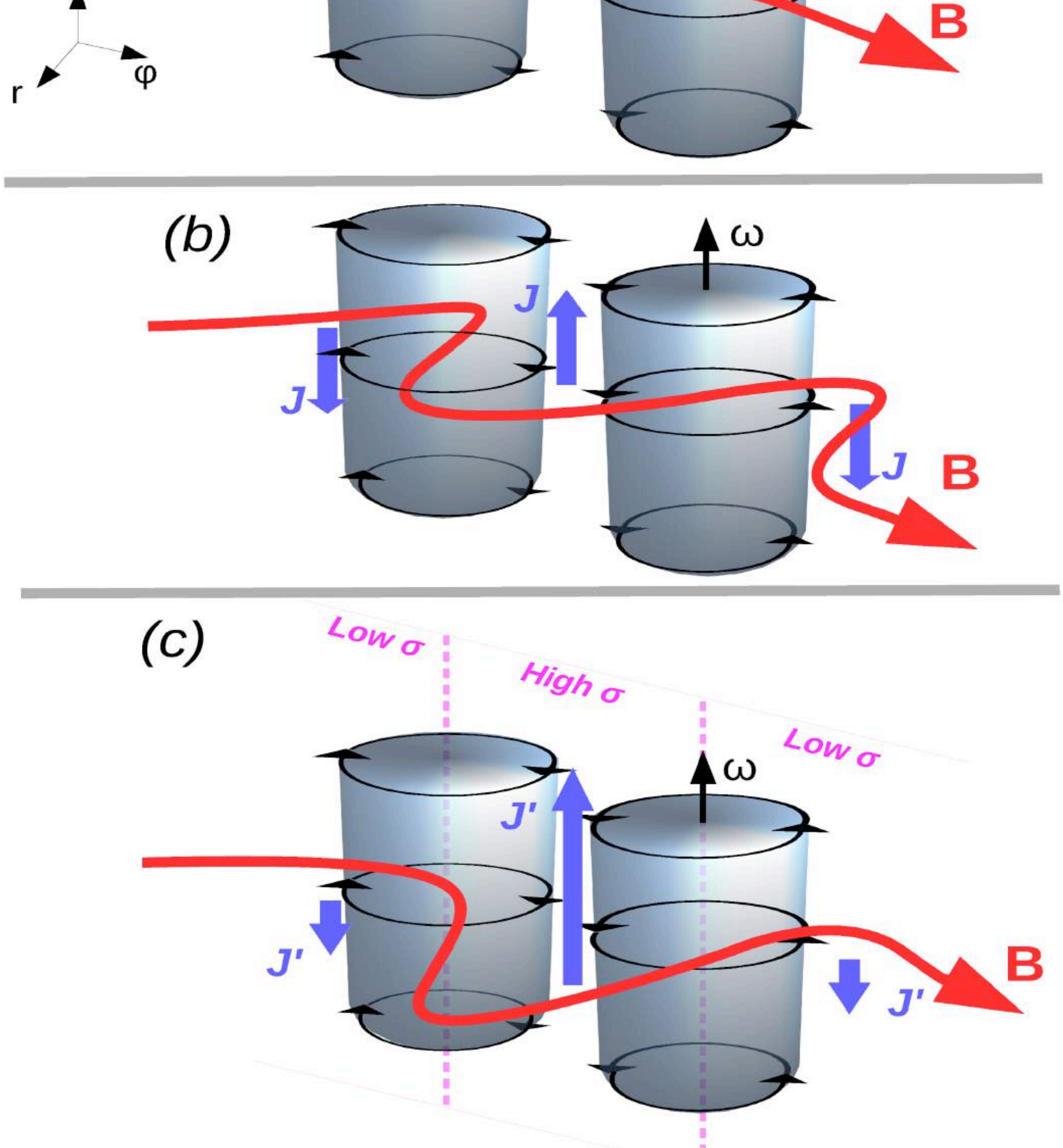
(c)
Low σ
High σ
Low σ
ω


r
φ
B
(b)
ω
J
J
J
B
(c)
Low σ
High σ
Low σ
ω
J'
J'
J'
B

Note that some antidynamo theorems do not apply

Dynamo with a planar flow!!

$$\mathbf{v} = (A\cos(ky)\sin(kz), B\cos(kx)\sin(kz), 0)$$

$$\delta\eta/\eta_0 = \delta(\cos(kz)(\sin(ky) - \sin(kx)))$$

We obtain

$$\langle \mathbf{v} \times \mathbf{b} \rangle = 0 \text{ and } \langle -\delta\eta\nabla \times \mathbf{b} \rangle = \delta/8\,(BBx,\ ABy,\ -(A+B)Bz)$$

For a large scale field such as $\exp\left(pt + iKz\right)$

$$p = \frac{|\delta K|\sqrt{AB}}{8} - \eta_0 K^2$$

Interest in the geo/stellar or galactic dynamo context?

Usually in MHD models of dynamos:

Consider uniform electrical conductivity

Fast rotation renders flows two dimensional,

Two dimensional flows are not dynamos.

The problem is complicated, DNS donnot observe dynamo at low Eckmann (fast rotation)

But:

If we consider varying electrical conductivity, two dimensionality is not a problem anymore!

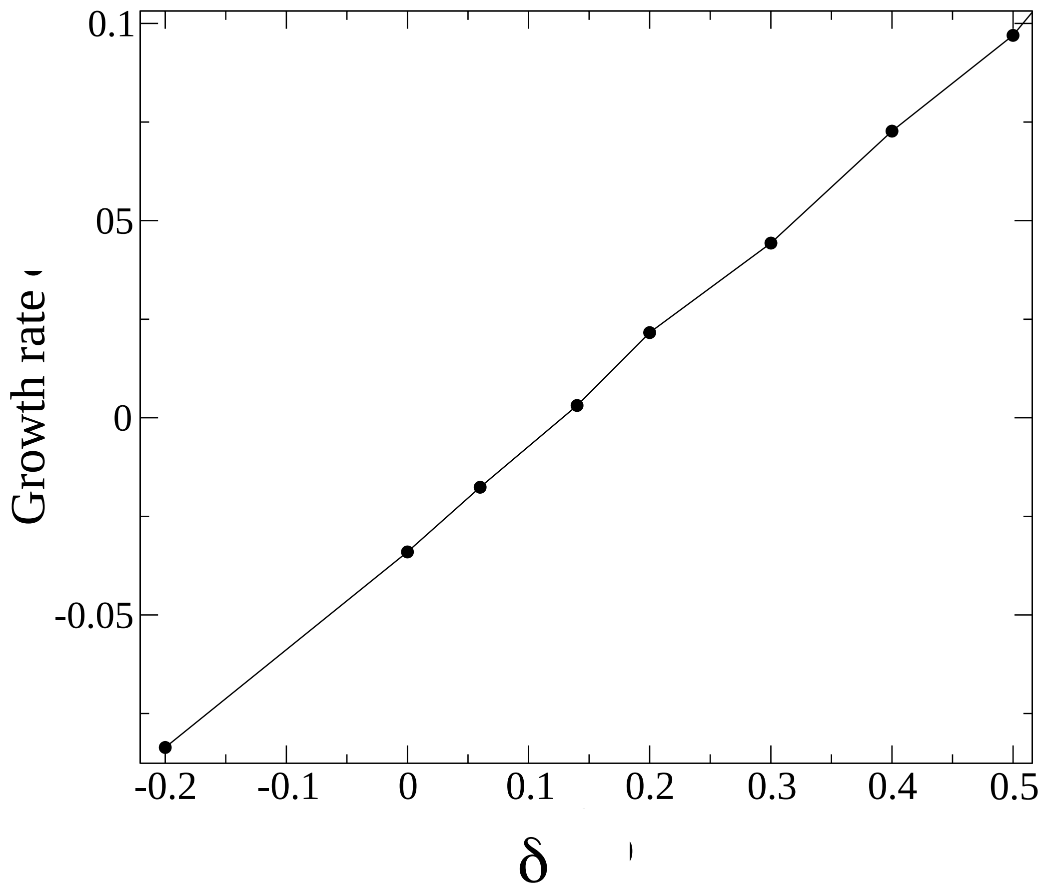
This seems nice for fastly rotating dynamos

Also, no need for helicity

C. Gissinger is running DNS of MHD with variable conductivity

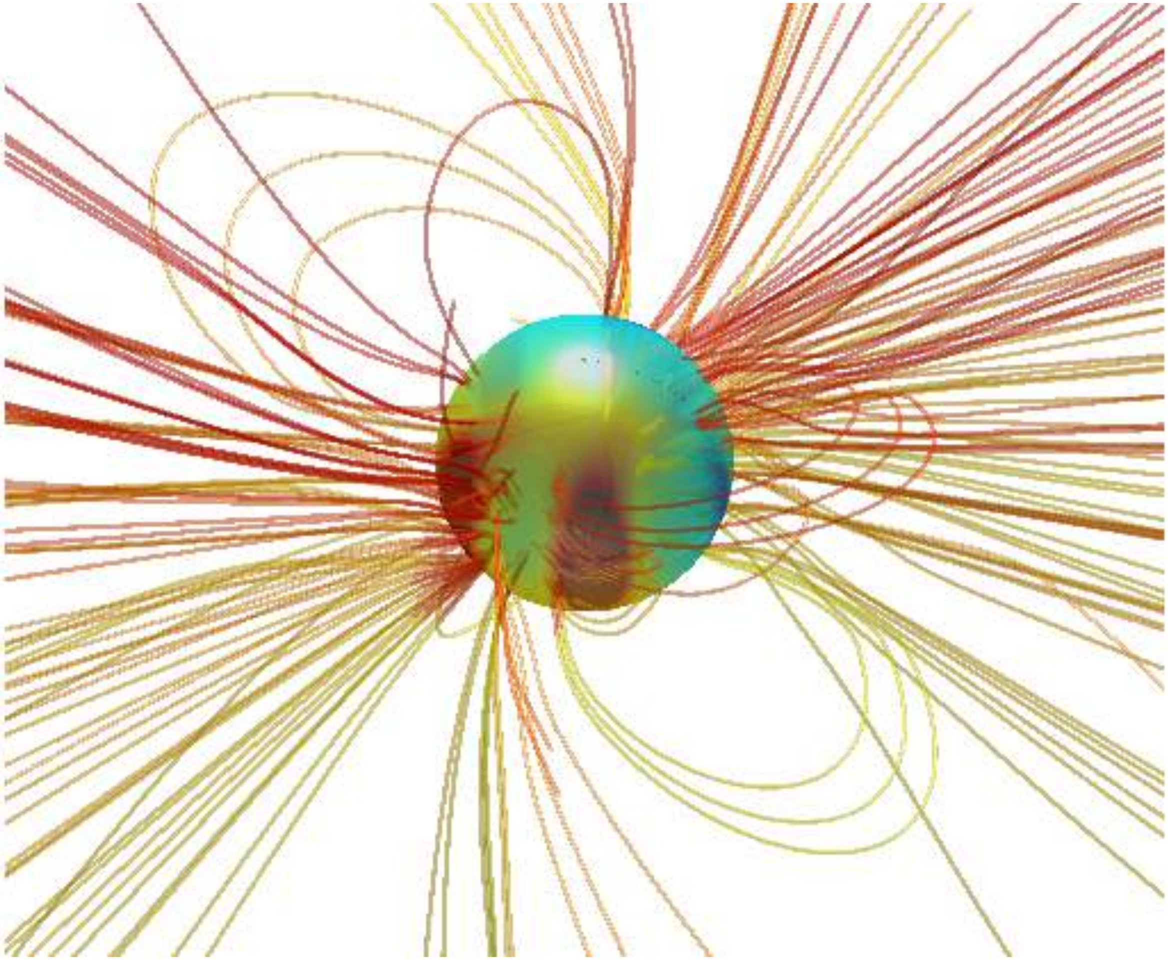
Results using Parody: Ek=1.e-3, Pm=8 et Ra/Rac=1.8

Growth rate of the transverse dipole is increased by conductivity variations



Shape of the unstable mode

Transverse dipole are more prone to dynamo



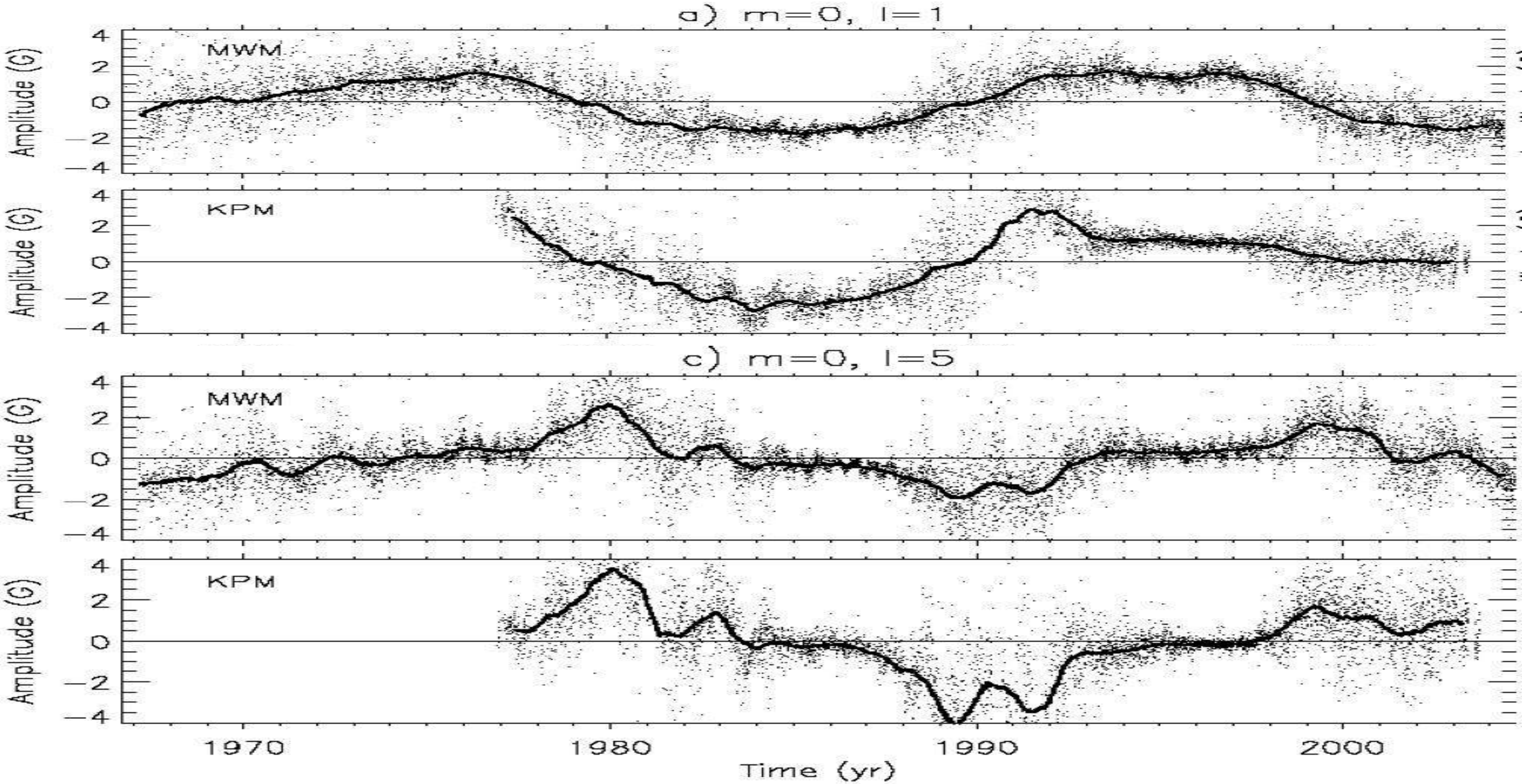
Questions:

How large are the variations of electrical conductivity in astrophysical dynamos?

For the Sun (or other stars)
Is there a second mode involved in the periodic dynamics.

Above the onset of the saddle-node bifurcation: a noisy oscillation

The Sun?



Knaack and Stenflo, A & A, 438 (2005)