CONSTRAINTS ON PRIMORDIAL MAGNETIC FIELDS WITH CMB ANISOTROPIES

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“Magnetic Fields in the Universe: from Laboratory and Stars to the primordial Structures”, Cargese, October 2015
OUTLINE

• WHY PRIMORDIAL MAGNETIC FIELDS

• Primordial magnetic fields and the cosmic microwave background anisotropies

• Magnetically induced perturbations

• Planck 2015 constraints part I: Likelihood

• Planck 2015 constraints part II: Non-Gaussianities

• Planck 2015 constraints part III: Faraday rotation

• Conclusions

• Future perspectives
The study of primordial magnetic fields is like a good red wine... it becomes more and more interesting with time!
First we had the observations of large scale magnetic fields in galaxies and clusters! The generation of such fields may benefit from the presence of initial -primordial- seed -see for example Riu 2011 for a review or Xu et al. 2012, Govoni et al. 2013, Marinacci et al. 2015, for example of MHD simulations of LSS-

Then, magnetic fields could be generated in the early universe:

- Or during recombination -Gopal and Sethi 2005, Ichiki et al. 2007, Fenu et al. 2011, Saga et al. 2015-

DIFFERENT GENERATION MECHANISMS LEAD TO DIFFERENT PMF CHARACTERISTICS

Now we have also large scale magnetic fields without an associated structure? May interact with the pairs produced by TeV photons from blazars and reduce the Gev flux. FERMI observations of the Blazar 1ES 0229+200 presents a lack of flux of GeV photons with respect to the predictions. A possible interpretation is the presence of a diffuse magnetic field -Neronov and Vovk 2010, Tavecchio et al. 2010, Tavecchio e al. 2010, Taylor et a Al. 2011, Neronov et al.2011, Neronov et al. 2013, Bonnoli et al. 2015-

LOWER LIMITS ON THE AMPLITUDE OF THE FIELDS

$10^{-18} - 10^{-15}$ Gauss
OUTLINE

• Why primordial magnetic fields

• **PRIMORDIAL MAGNETIC FIELDS AND THE COSMIC MICROWAVE BACKGROUND ANISOTROPIES**

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The Cosmic Microwave Background (CMB) is the relic radiation of the Big Bang. It was emitted when the matter decoupled from radiation, thanks to the recombination of neutral hydrogen, 380000 yrs after the Big-Bang.
CMB anisotropies are the footprints of cosmological perturbations.

PMF modify the evolution of cosmological perturbations and have a direct effect on CMB anisotropies.

PMF modelled as a stochastic background have a fully non-Gaussian impact on CMB anisotropies.

PMF induce a Faraday rotation of the CMB anisotropies in polarization.

CMB anisotropies are polarized thanks to the Compton scattering between photons and electrons before recombination.

CMB anisotropies thanks to the variety of probes in a single observable, are one of the best laboratory to investigate PMF.

Primordial perturbations in the standard cosmological model are Gaussian.

PMF affect CMB anisotropies in 3 ways:
Temperature anisotropies are expanded in spherical harmonics:

$$\Delta T(\vec{x}, \hat{n}, \tau) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm}(\vec{x}, \tau) Y_{lm}(\hat{n})$$

The temperature anisotropy distribution is characterized by the two point correlation function: **Angular Power Spectrum**

$$C_l = \frac{1}{2l+1} \sum_{m} \langle a^*_{lm} a_{lm} \rangle$$

**Planck 2015 Temperature APS**

- **Large angular scales**: scales outside the horizon at recombination. Only gravity.
- **Intermediate scales**: Acoustic oscillations of the photon baryon fluid. Dark matter potential wells vs radiation pressure.
- **Small angular scales**: Perturbations are suppressed by Silk damping. Data contamination by astrophysical signals.
The CMB is linearly polarized, but polarization is not analysed using the Q and U Stokes parameters but their combinations E-mode and B-mode.

Primary B-mode polarization is not generated by scalar perturbations but only by vector and tensors. Odd cross correlators Temperature-B-mode and E-B are generated only by some extended cosmological models.

Credits Wayne Hu
http://background.uchicago.edu/

Planck 2015 E-mode

Planck 2015 cross correlation Temperature-E-mode

Planck+BICEP 2+KECK B-mode (blue points, black are the original BKxBK data)
• Why primordial magnetic fields

• Primordial magnetic fields and the cosmic microwave background anisotropies

• **MAGNETICALLY INDUCED PERTURBATIONS**

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An homogenous field would break the assumption of isotropy and would not be supported in a LFRW universe. **PMF modelled as a stochastic background.** We can neglect all the contributions to the background.

**PMF source all types of perturbations:**

**Scalar**

- Standard perturbations in the energy density and pressure of the cosmological fluid.
- These perturbations grow originating the large scale structures of the universe

**Vector**

- Represent the vortical motions of matter in the plasma.
- The vorticity decreases in a LFRW universe. The standard vector mode sourced by neutrinos is in fact a decaying mode.

**Tensor**

- Tensor perturbations are traceless and transverse metric perturbations.
- Tensor modes are a key prediction of many inflationary models. Their observation is one of the crucial point of cosmology.

**Credits** Wayne Hu
http://background.uchicago.edu/
Magnetically induced perturbations have just a very small dedicated literature...
This is only a small outdated subset...

SCALAR
- Adams et al. 1996
- Koh & Lee 2000
- Grasso & Rubinstein 2001
- Giovannini 2004, 2006, 2006/2
- Yamazaki et al. 2005, 2006
- Kahniashvili & Ratra 2007
- Giovannini & Kunze 2008
- Finelli et al. 2008
- Bonvin & Caprini 2010
- ........
- Giovannini 2004
- Durrer 2007
- Subramanian 2010
- Caprini 2011
- Durrer and Neronov 2014

VECTOR & TENSOR
- Caprini & Durrer 2003
- Subramanian et al. 2003
- Kojima et al. 2008
- Lewis 2004
- Subramanian & Barrow 2002
- Durrer et al. 2000
- Lewis 2000
- Mack et al. 2002
- Paoletti et al. 2009
- ....................
- Giovannini & Kunze 2008/2, 2008/3
- Kahniashvili et al. 2010
- Kojima & Ichiki 2009
- Shaw & Lewis 2010, 2012
- Paoletti & Finelli 2011, 2013
- Planck Coll. 2013
- Ballardini, Finelli, Paoletti 2015
- Planck Coll. 2015
- ....................

REVIEWS
- Caprini & Durrer 2003
- Subramanian et al. 2003
- Kojima et al. 2008
- Lewis 2004
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- Giovannini & Kunze 2008/2, 2008/3
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- Shaw & Lewis 2010, 2012
- Paoletti & Finelli 2011, 2013
- Planck Coll. 2013
- Ballardini, Finelli, Paoletti 2015
- Planck Coll. 2015
- ....................

ALL
Cosmological perturbations are described by the coupled system of Einstein equations for metric perturbations and the Boltzmann equations for the fluid perturbations.

PMF are an additional component to the plasma like neutrinos, photons, baryons etc. But behave in a different way because their contribution to the background is negligible.

PMF energy momentum tensor enters as an additional source term in the perturbed Einstein equations and the Lorentz force modifies the baryon evolution.

$$\delta G_{\mu\nu} = 8\pi(\delta T_{\mu\nu} + \tau_{\mu\nu}^{PMF})$$

Lorentz force term in baryons equations

$$\nabla_\mu \delta T^{\mu\nu} \propto F^{\mu\nu} J_\mu$$

Before recombination baryons and photons are coupled by the scattering and therefore there is an indirect effect of the Lorentz force also on photons.
PMF MODEL AND ENERGY MOMENTUM TENSOR

Two point correlation function
\[
\langle B_i(k)B_j^*(k') \rangle = \frac{(2\pi)^3}{2} \delta^{(3)}(k - k') \left[ (\delta_{ij} - \hat{k}_i \hat{k}_j)P_B(k) + i \epsilon_{ijl} \hat{k}_l P_H(k) \right]
\]

Power-law power spectrum for both helical and non-helical parts
- Non-helical part
  \[ P_B(k) = A_B k^{n_B} \]
- Helical part
  \[ P_H(k) = A_H k^{n_H} \]

Magnetized perturbations survive silk damping but are suppressed on smaller scales. 
*Subramanian and Barrow 1997, Jedamzik et al 1997*

\[
k_D = (2.9 \times 10^4) \frac{2}{n_B+5} \left( \frac{B_\lambda}{n\text{Gauss}} \right)^{-2} \left( k_\lambda \frac{1}{1\text{Mpc}^{-1}} \right)^{n_B+3} \Omega_b h^2 \text{Mpc}
\]

We modelled this damping with a sharp cut off in the PMF spectra

\[
\langle B_i^*(\vec{k})B_j(\vec{k}') \rangle = \begin{cases} 
\delta^3(\vec{k} - \vec{k}') (\delta_{ij} - \hat{k}_i \hat{k}_j) \frac{P_B(k)}{2} & \text{for } k \leq k_D \\
0 & \text{for } k > k_D
\end{cases}
\]

In the primordial universe we can assume the MHD limit and neglect backreaction of the fluid onto the fields

\[
\rho_B(x, \tau) = \frac{\rho_B(x)}{a^4(\tau)} \rightarrow B(x, \tau) = \frac{B(x)}{a^2(\tau)}
\]

Where \(a(t)\) is the scale factor
The evolution of cosmological perturbations is performed in the Fourier space. We have different ways to parametrize the fields in Fourier.

RMS OF THE FIELDS
\[ \langle B^2(x) \rangle = \int_{k < k_D} d^3k P_B(k) = \frac{4\pi A}{n_B + 3} \frac{k_{*D}^{n_B+3}}{k_*^{n_B}} \]

SMOOTHED FIELDS
Used to have a reference scale, usually 1 Mpc
\[ \langle B^2_\lambda(x) \rangle = \int d^3k e^{-\lambda^2 k^2} P_B(k) = 2\pi A \frac{\Gamma\left(\frac{n_B}{2} + 3\right)}{\lambda^{n_B+3}} \]

HELICAL COMPONENT
\[ \langle B^2_\lambda \rangle = A \int_0^\infty \frac{dk}{2\pi^2} e^{-\lambda^2 k^2} |P_H(k)| = \frac{|A_H|}{4\pi^2 \lambda^{n_H+3}} \Gamma\left(\frac{n_H + 4}{2}\right) \]

n_B > -3 to avoid divergences

The energy momentum tensor power spectrum in Fourier space is given by very complex convolutions of the fields...

We need to compute the scalar, vector an tensor projections

The resulting spectra (Finelli et al. 2008, Paoletti et al. 2009) have very complex formulae

In the following we show a couple of the simpler results
Magnetized CMB angular power spectrum strongly depends on the behaviour of the PMF EMT components in the infrared limits $k \to 0$

**INFRARED COMMON BEHAVIOUR**
Determines the CMB anisotropy angular power spectra

$$n_B > -3/2 \rightarrow \text{white noise}$$

$$n_B < -3/2 \rightarrow k^{(2n_B+3)}$$

$$k^3 |\rho(k)|^2 \text{ in units of } \langle B^2 \rangle^2/(4\pi)^4 \left( \langle B^2 \rangle^2/(4\pi)^4 \right)$$

$$|\rho_B(k)|^2_{n_B=2} = \frac{A^2 k_D^7}{512 \pi^4 k_s^8} \left[ \frac{4}{7} - \frac{8 k^2}{15} + \frac{11 k^7}{2240} \right],$$

$$|\Pi_B^{(\nu)}(k)|^2_{n_B=2} = \frac{A^2 k_D^7}{256 \pi^4 k_s^8} \left[ \frac{4}{15} - \frac{5 k}{12} + \frac{7 k^3}{960} - \frac{7 k^5}{1920} \right],$$

$$|\Pi_B^{(T)}(k)|^2_{n_B=2} = \frac{A^2 k_D^7}{256 \pi^4 k_s^8} \left[ \frac{8}{15} - \frac{16 k^2}{15} - \frac{13 k^5}{480} + \frac{11 k^7}{1920} \right].$$

*Paoletti et al. 2009*
INITIAL CONDITIONS

Magnetically induced perturbations does not only come with different modes -scalar, vector and tensors- but also with different initial conditions.

Initial conditions are the status of cosmological perturbations when we start to evolve the equations: they are the solutions of the equations at early times (for Einstein Boltzmann codes is after neutrino decoupling) on large wavelength.

Different initial conditions source different perturbations

- **Compensated:** Magnetically induced modes which are sourced by PMF energy momentum tensor after neutrino decoupling. The «compensated» definition comes from the compensation of magnetic terms by the fluid perturbations (Giovannini 2004, Lewis 2004, Finelli et al. 2008, Paoletti et al. 2009, Shaw & Lewis 2010).

- **Passive:** This mode is generated prior to the neutrino decoupling when the anisotropic stress of PMF has no counterpart in the fluid. This uncompensated source gives rise to an extra solution, logarithmic in time. After neutrino decoupling with the rise of their anisotropic stress, which compensates the PMF one, this solution no longer exists. But it leaves a footprint in the form of an offset in the amplitude of the inflationary mode for scalar and tensor perturbations (Lewis 2004, Shaw and Lewis 2010).

- **Inflationary:** This mode is strictly related to inflationary generated fields and is strongly dependent on the generation mechanism of the fields (Bonvin et al. 2011, 2013).
MAGNETICALLY INDUCED ANISOTROPIES

- Einstein equations for metric perturbations
  + Boltzmann equations for fluid component perturbations

CAMB = public Einstein Boltzmann code which evolves the equations and compute the CMB anisotropy angular power spectra –Lasenbly, Challinor, Lewis 2000–

- PMF contributions to the equations
- PMF energy momentum tensor components
- Magnetic initial conditions

CMB magnetically induced angular power spectra for scalar, vector and tensor modes
Dotted magnetized scalar mode  
Solid primary scalar mode  
Standard non-magnetic modes

NON-HELICAL COMPENSATED MAGNETICALLY INDUCED ANGULAR POWER SPECTRA

The dominant PMF contribution is given by the vector mode on small angular scales!

Planck 2015 Results XIX

$B_{1\text{Mpc}} = 4.1 \, nG$

$n_B = -1$
Passive tensor

APS dependence on the PMF spectral index

Behaviour driven by the PMF EMT spectrum:

- $n_B > -3/2 \rightarrow$ white noise
- Uniform spectral shape with the only variations in the amplitude. It is given by the white noise spectra of PMF EMT
- $n_B < -3/2 \rightarrow k^{(2n_B+3)}$
- Show a spectral shape which tilts accordingly to the infrared dominated behaviour of the PMF EMT

$B_{1Mpc} = 4.1 \, nG$

$n_B = -2.9$

Planck 2015 Results XIX
HELICAL MAGNETIZED CMB ANGULAR POWER SPECTRA

The helical component gives an extra term in the PMF EMT. This term has a contribution which diminishes the amplitude of the magnetically induced APS.

The antisymmetric part of the helical component EMT generates non-zero ODD CMB cross correlator TB and EB.

\[ n_B = n_H \]
\[ A_B = A_H \]

\[ B_{1\text{Mpc}} = 4.1 \ nG \]
\[ n_B = -1 \]
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  • Planck 2015 constraints part II: Non-Gaussianities
  
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The predictions for the CMB angular power spectra are used to derive the constraints on PMF amplitude.

We explore the cosmological parameter space with the Markov Chain MonteCarlo code Cosmomc (Bridle & Lewis 2002), extended in order to include PMF contributions (Paoletti & Finelli 2010).

In addition to the standard six standard model parameters (baryon density, cold dark matter density, angular diameter distance horizon at recombination, spectral index, amplitude of primordial fluctuations, optical depth) we vary the PMF amplitude and spectral index (for the case which include the passive mode also the additional parameter \((\tau_D/\tau_V)\)).

We used the Planck 2015 likelihood with different combinations:

- **Planck TT / Planck TTTEEE** indicates the high-ell Planck likelihood, with either temperature only (TT) or temperature plus polarization (TTTEEE)
- **LowP** indicates the Planck low-ell likelihood based on the component separated Commander map for temperature and the LFI 70GHz maps cleaned with 30 GHz and 353 GHz for the polarization.
CONSTRAINTS WITH PLANCK LIKELIHOOD I

The marginalized posterior distributions are the parameter distributions marginalized over all the other parameters.

### SPECTRAL INDEX

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<th>$n_B$</th>
<th>$B_{1,\text{Mpc}}$ [nG]</th>
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<tbody>
<tr>
<td>$n_B &gt; 0$</td>
<td>$B_{1,\text{Mpc}} &lt; 0.55$</td>
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<tr>
<td>$n_B = 2$</td>
<td>$B_{1,\text{Mpc}} &lt; 0.01$</td>
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<tr>
<td>$n_B = -2.9$</td>
<td>$B_{1,\text{Mpc}} &lt; 2.1$</td>
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<tr>
<th>Likelihood Combination</th>
<th>$B_{1,\text{Mpc}}$ [nG]</th>
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<tr>
<td>$TT, TE, EE + \text{lowP}: C$</td>
<td>$&lt; 4.4$</td>
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<tr>
<td>$TT + \text{lowP}: C$</td>
<td>$&lt; 4.4$</td>
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<tr>
<td>$TT, TE, EE + \text{lowP}: C + P$</td>
<td>$&lt; 4.5$</td>
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<tr>
<td>$TT + \text{lowP}: C + P$</td>
<td>$&lt; 4.5$</td>
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<tr>
<td>$TT + \tau_{\text{reion}} \text{ prior: C+P}$</td>
<td>$&lt; 4.4$</td>
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Strong degeneracy between the amplitude and the spectral index.
A degeneracy happens when we can realize the same angular power spectra with different combinations of the amplitude and spectral index. It is characterized by diagonal shapes in the 2-D plots.

The impact of PMF is on small angular scales where the CMB is contaminated by astrophysical residuals. The Planck likelihood includes the treatment of this contamination with the marginalization over the dominant contributions.
We note how PMF show a degeneracy with the Poissonian terms (due to the random distribution on the sky of the residual, under detection threshold, point sources).
CONSTRAINTS FOR HELICAL FIELDS

MAXIMALLY HELICAL

The constraint on PMF amplitude with an helical component is

$$B_{1 \text{Mpc}} < 5.6 \text{ nG}$$

The constraint is lower with respect to the non-helical case because we have shown that the helical contribution reduces the amplitude of the magnetically induced spectra.

The constraints are derived with the Planck TT and lowP likelihood and they include only the even-power spectra.

JOINT PLANCK+BICEP 2/KECK Array

No significant evidence of PMF in the Cross BICEP2/KECK/Planck data

$$B_{1 \text{Mpc}} < 4.7 \text{ nG}$$
The presence of PMF modifies the ionization history. This is due to the injection of energy into the plasma caused by the dissipation of the PMF. In particular we have two main mechanisms (Sethi & Subramanian 2005, Chluba et al. 2015, Kunze & Komatsu 2015):

**Ambipolar Diffusion**  ←  **MHD Decaying Turbulence**

This has an impact on the ionization fraction and the electron temperature.

The change in the ionization history is reflected on the CMB angular power spectra. **Planck 2015 likelihood using only this contribution gives constraints to less than a nG**
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PMF modelled as a stochastic background have a fully non-Gaussian impact on CMB anisotropies. PMF generates non-zero three point correlation function (bispectrum) and four point correlation function (trispectrum)

The constraints derived with the non-Gaussianity measurements are complementary to the ones derived with the Planck likelihood.

The angular power spectrum is the two point correlation function and it depends on the fourth power of the fields. 

Similarly the magnetically induced bispectrum, which is the three point correlation function, depends on the sixth power of the fields!

As for the two point correlation function also for non-Gaussianity analysis we can consider different initial conditions and different modes.

• Brown & Crittenden 2005- > general treatment of magnetic NG
• Brown 2008- > Ph.d Thesis
• Seshadri & Subramanian 2009- > Bispectrum of scalar magnetized mode
• Caprini, Finelli, Paoletti & Riotto 2009- > Bispectrum of scalar magnetized mode
• Trivedi, Subramanian & Seshadri 2010- > Bispectrum from magnetic scalar passive mode
• Shiraishi et Al. 2011- > Bispectrum from vector magnetized mode
• Shiraishi et Al. 2011/2 - > Bispectrum and constraints from magnetized passive tensor mode
• Trivedi, Subramanian & Seshadri 2011 - > Trispectrum from magnetic scalar mode
• Shiraishi et Al.2012->Bispectrum from scalar & tensor passive modes
• ........................................

In Planck 2015 results XIX we have considered three cases:

• Tensor passive bispectrum
• Anisotropic bispectrum for passive modes
• Compensated scalar bispectrum
The tensor passive mode is the dominant contribution to the large scale angular power spectrum for scale invariant PMF ($n_B=-2.9$).

We have considered the magnetized passive tensor bispectrum for $l<500$ and the squeezed limit configuration in which the passive bispectrum is amplified.

$$\ell_1 \ll \ell_2 \approx \ell_3$$

$$A_{\text{bis}} \equiv \left( \frac{B_{1 \text{ Mpc}}}{3 \text{ nG}} \right)^6 \left[ \frac{\ln(\tau_v/\tau_B)}{\ln(10^{17})} \right]^3,$$


The limits on the bispectrum amplitude can be translated into limits for the fields

SMICA FG cleaned maps T and E for PMF generated at the Grand Unification scale with $n_B=-2.9$

$$B_{1 \text{ Mpc}} < 2.8 \text{ nG}$$
ANISOTROPIC BISPECTRUM

Considering the curvature perturbations induced by passive modes

\[ \zeta_k \approx 0.9 \ln \left( \frac{\tau_y}{\tau_B} \right) \frac{1}{4\pi \rho_{\gamma,0}} \sum_{ij} \left( k_i k_j - \frac{1}{3} \delta_{ij} \right) \int \frac{d^3 k'}{(2\pi)^3} B_i(k') B_j(k - k') . \]

PMF produce non-vanishing bispectrum of direction-dependence

\[ \langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \rangle = \sum_L c_L \left( P_L(\hat{\vec{k}}_1 \cdot \hat{\vec{k}}_2) P_\phi(k_1) P_\phi(k_2) + 2 \text{ perm} \right) \]

The zeroth and the second expansion coefficients are related to the amplitude of magnetic fields:

\[ c_0 \approx -2 \times 10^{-4} \left( \frac{B_{1 \text{Mpc}}}{\text{nG}} \right)^6 , \]

\[ c_2 \approx -2.8 \times 10^{-3} \left( \frac{B_{1 \text{Mpc}}}{\text{nG}} \right)^6 \]

**Constraints on the amplitude for \( B_{1 \text{Mpc}} \) [nG] with \( n_B = -2.9 \) generated at the GUT scale for the four component separation maps available in Planck 2015**

<table>
<thead>
<tr>
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<th>SMICA</th>
<th>NILC</th>
<th>SEVEM</th>
<th>Commander</th>
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<tbody>
<tr>
<td>( B_{1 \text{Mpc}}/\text{nG} ) . . .</td>
<td>&lt; 4.5</td>
<td>&lt; 4.9</td>
<td>&lt; 5.0</td>
<td>&lt; 5.0</td>
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SCALAR MAGNETIZED BISPECTRUM

We derived the analytical magnetized compensated scalar bispectrum on large angular scales. The temperature anisotropy for PMF can be written as

\[
\frac{\Theta_{\ell}^{(0)}(\eta_0, \mathbf{k})}{2\ell + 1} = \frac{\alpha}{4} \Omega_B(\mathbf{k}) j_\ell(k(\eta_0 - \eta_{\text{dec}})),
\]

The magnetized bispectrum depends on the magnetic energy density bispectrum

\[
\langle \rho_B(\mathbf{k}) \rho_B(\mathbf{q}) \rho_B(\mathbf{p}) \rangle = \frac{1}{(8\pi)^3} \int \frac{d^3 \tilde{k} d^3 \tilde{q} d^3 \tilde{p}}{(2\pi)^9} \langle B_i(\tilde{k}) B_i(\mathbf{k} - \tilde{k}) B_j(\tilde{q}) B_j(\mathbf{q} - \tilde{q}) B_l(\mathbf{p}) B_l(\mathbf{p} - \mathbf{\tilde{p}}) \rangle.
\]

Contrary to the passive case for compensated mode there is no a-priori dominant geometrical configuration

By the comparison of the bispectrum and the spectrum it is possible to derive an effective \( f_{\text{nl}} \) in the local configuration to be compared with the measured one (SMICA KSW) to constrain PMF

\[
f_{\text{NL}}^{\text{eff}} \approx \frac{3\pi^9}{2304} \frac{\alpha^3}{A^2} \frac{n_B(n_B + 3)^2}{2n_B + 3} \frac{\langle B^2 \rangle^3}{\rho_{\text{rel}}^3} \approx 1.2 \times 10^{-3} (n_B + 3)^2 \left( \frac{\langle B^2 \rangle}{(10^{-9} \text{ G})^2} \right)^3.
\]

\[
B_{1 \text{ Mpc}} < 3.0 \text{ nG (95\% CL, } n_B = -2.9\text{)}
\]
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FARADAY ROTATION

The presence of PMF induces a rotation of the polarization plane of CMB anisotropies rotating $E$-mode polarization into $B$-mode and vice versa. The Faraday depth is given by

$$\Phi = K \int n_e(x, n) B_{\parallel}(x, n) \, dx.$$  

**B and E mode polarization rotated spectra**

$$C_{\ell}^{BB} = N_{\ell}^2 \sum_{\ell_1, \ell_2} \frac{(2\ell_1 + 1)(2\ell_2 + 1)}{4\pi(2\ell + 1)} N_{\ell_2}^2 K(\ell, \ell_1, \ell_2)^2 C_{\ell_2}^{EE} C_{\ell_1}^{\alpha} \left( C_{\ell_0 \ell_2 \ell_0}^{00} \right)^2$$

$$C_{\ell}^{EE} = N_{\ell}^2 \sum_{\ell_1, \ell_2} \frac{(2\ell_1 + 1)(2\ell_2 + 1)}{4\pi(2\ell + 1)} N_{\ell_2}^2 K(\ell, \ell_1, \ell_2)^2 C_{\ell_2}^{BB} C_{\ell_1}^{\alpha} \left( C_{\ell_1 \ell_0 \ell_0}^{00} \right)^2$$

$$C_{\ell}^{\Phi} \simeq \frac{9\ell(\ell + 1)}{(4\pi)^3 e^2} \frac{B_\lambda^2}{\Gamma(n_B + 3/2)} \left( \frac{\lambda}{\eta_0} \right)^{n_B + 3} \int_0^{x_D} dx x^{n_B} f_\ell^2(x).$$

**Strong frequency dependence! Lower frequencies are more affected by Faraday rotation**

The $EE$ mode from Planck 70 GHz ($2 < \ell < 29$) spectrum has been used to derive the expected $BB$ rotated mode. Comparison with measured $B$-modes at 70 GHz computing the minimum $\chi^2$. 
Estimate of the Galactic contribution, subdominant for our data

$B_{1\text{ Mpc}} < 1380 \text{ nG}$
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• Future perspectives
CONCLUSIONS

Ever increasing accuracy of cosmological data allows to strongly constraints PMF amplitude and in particular CMB data have been proven to be one of the best laboratory to investigate and constrain PMF.

A stochastic background of PMF leaves different peculiar imprints on CMB anisotropies through scalar, vector and tensor contributions both in temperature and polarization.

- At the CMB angular power spectrum level the stronger contribution is given by magnetically induced vector perturbations on small angular scales. For scale invariant spectral index it is relevant also the contribution of the passive tensor mode on large angular scales. A strong contribution is provided also by the impact of PMF on the ionization history. We have shown the constraints derived with the Planck 2015 likelihood are at the few nG level.

- A stochastic background of PMF has a fully non-Gaussian impact on CMB anisotropies generating non-zero higher statistical moments. In particular, PMF generates a non-zero bispectrum with different modes and initial conditions. Using different bispectra and different techniques Planck 2015 has show that non-Gaussianity constraints are very competitive with likelihood ones.
PMF induce a Faraday rotation of the CMB anisotropy in polarization generating a B-mode polarization from the primary E-mode. Using the BB-spectrum available from the 70GHz Planck likelihood it is possible to give constraints on PMF. These constraints are based on a very limited range of multipoles where the signal is subdominant and therefore are larger with respect to the other methods.

<table>
<thead>
<tr>
<th>Model/Dataset/Method</th>
<th>nG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planck TT+lowP</td>
<td>$B_{1\text{Mpc}} &lt; 4.4$</td>
</tr>
<tr>
<td>Planck TT,TE,EE+lowP</td>
<td>$B_{1\text{Mpc}} &lt; 4.4$</td>
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<tr>
<td>$n_B &gt; 0$</td>
<td>$B_{1\text{Mpc}} &lt; 0.55$</td>
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<tr>
<td>$n_B = 2$</td>
<td>$B_{1\text{Mpc}} &lt; 0.01$</td>
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<tr>
<td>$n_B = -2.9$</td>
<td>$B_{1\text{Mpc}} &lt; 2.1$</td>
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<tr>
<td>Helical PMF</td>
<td>$B_{1\text{Mpc}} &lt; 5.6$</td>
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<tr>
<td>Planck+BICEP 2/KECK ARRAY</td>
<td>$B_{1\text{Mpc}} &lt; 4.7$</td>
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<tr>
<td>Impact on the ionization history</td>
<td>$B_{1\text{Mpc}} &lt; 1$</td>
</tr>
<tr>
<td>Passive tensor mode bispectrum $n_B = -2.9$</td>
<td>$B_{1\text{Mpc}} &lt; 2.8$</td>
</tr>
<tr>
<td>Passive anisotropic bispectrum $n_B = -2.9$</td>
<td>$B_{1\text{Mpc}} &lt; 4.5$</td>
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<tr>
<td>Scalar compensated bispectrum $n_B = -2.9$</td>
<td>$B_{1\text{Mpc}} &lt; 3.0$</td>
</tr>
<tr>
<td>Faraday rotation</td>
<td>$B_{1\text{Mpc}} &lt; 1380$</td>
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</tbody>
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OUTLINE

• Why primordial magnetic fields

• Primordial magnetic fields and the cosmic microwave background anisotropies

• Magnetically induced perturbations

• Planck 2015 constraints part I: Likelihood

• Planck 2015 constraints part II: Non-Gaussianities

• Planck 2015 constraints part III: Faraday rotation

• Conclusions

• FUTURE PERSPECTIVES
FUTURE PERSPECTIVES

TEMPERATURE
Current and future ground experiments will be targeted on small angular scales where the impact of PMF is larger (e.g. SPT, ACT, S4 etc.)

POLARIZATION
Planck 2016 together with ground based experiments (BICEPP/KECK, SPTPOL, ACTPOL, PolarBear etc.) will provide accurate polarization measurements. Especially BB and TB/EB will be crucial to constrain PMF

NON GAUSSIANITIES
Full sky coverage requires space missions. Planck 2016 will provide accurate non-Gaussianity measurements in polarization. In the future hopes are for LiteBird, Pixie and maybe an ESA mission?

SPECTRAL DISTORTION
PMF dissipating around recombination may generate spectral distortions which can represent an independent way to constrain PMF. Possible hopes for a Pixie or Pixie like experiments.

BEYOND CMB
• PMF have an impact on the structure formation and therefore LSS data of future surveys and EUCLID will provide new constraints on PMF.
• PMF may have an impact on reionization..therefore we have future perspectives for the 21 cm measurements.
• SKA will have the resolution to investigate CMB Faraday rotation, together with providing a wide landscape of cosmic magnetism.
• CTA dedicated observations of targeted blazars to investigate diffuse magnetic fields in voids will help to support the hypothesis of magnetic fields diffuse in void regions

Plus a lot of theoretical improvements expected in the next years...
PART OF THE WORK PRESENTED HAVE BEEN DONE IN THE FRAMEWORK OF THE PLANCK COLLABORATION

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.