Energy dissipation in incompressible turbulence with ambipolar diffusion

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Incompressible AD MHD

Why is it needed?

- **Incompressible:** There are regions in diffuse clouds where $M_s \simeq 0.1$ and the dynamics are incompressible
- **AD**:
  - In diffuse clouds, ionization fraction can be quite low ($x_i \lesssim 10^{-4}$), but still ion-neutral friction (AD) is an important process
  - represents an important, independent dissipation process
- **What's new?**
  - Study of AD in a 3D turbulent flow with explicit, physical dissipation

Incompressible ambipolar diffusion (AD) MHD

Single-fluid approximation

- Mass conservation: $\nabla \cdot \mathbf{u} = 0$
- Momentum conservation (Alfvén velocity $\mathbf{b} = \mathbf{B}/\sqrt{4\pi\rho}$)

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} = -\nabla\bar{p} + \mathbf{j} \times \mathbf{b} + Re^{-1}\nabla^2\mathbf{u}, \quad \bar{p} = \frac{p}{\rho},\ \mathbf{j} = \nabla \times \mathbf{b}$$

- No magnetic monopoles: $\nabla \cdot \mathbf{b} = 0$
- Induction equation + ambipolar diffusion

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{b}) + Re_a^{-1}\nabla \times \big((\mathbf{j} \times \mathbf{b}) \times \mathbf{b}\big) + Re_m^{-1}\nabla^2\mathbf{b}$$

- AD term is **dissipative**, but **non-reconnecting** - $\mathbf{b}$ is frozen in the ion velocity field $\mathbf{u} + Re_a^{-1}\mathbf{j} \times \mathbf{b}$

The AD length scale

- AD Reynolds number: $Re_a = l/(u\gamma\rho_i)$
- Dimensional comparison of Fourier amplitudes of $\mathbf{u} \times \mathbf{b}$ and $Re_a^{-1}(\mathbf{j} \times \mathbf{b}) \times \mathbf{b}$
- Critical wavenumber: $k_a = Re_a\sqrt{\langle u^2\rangle}/\langle b^2\rangle$
- Length scales below $l_a = 2\pi/k_a$, AD should be effective

Spectral method: Parallel spectral code ANK

http://lerma.obspm.fr/~momferratos/ank.html

- The fields are defined on a Cartesian grid of equidistant points and are represented by truncated Fourier series (fast Fourier transform using the FFTW library)

$$\mathbf{u}(\mathbf{x}) = \sum_{\mathbf{k} \in C_k} \mathbf{u}(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{x})$$

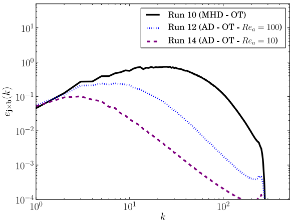
  where C_k is the discrete grid of wave-vectors
- **Boundary conditions:** periodic in all directions
- Advanced dealiasing, polyhedral truncation $\rightarrow$ increased small-scale accuracy
- **Advantages**: Negligible numerical dissipation, high accuracy in the small scales, incompressibility enforced naturally to machine precision
- **Drawbacks**: Cannot model shocks because of the Gibbs phenomenon, FFT communication is expensive

Incompressible AD MHD

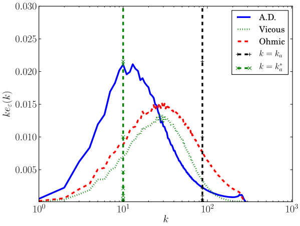
Alignment of j and b

- AD dissipation: $\varepsilon_a \propto (\mathbf{j} \times \mathbf{b})^2 = |\mathbf{j}|^2 |\mathbf{b}|^2 \sin^2 \theta$
- Spectra of **j** and **b** are almost unchanged
- **The AD length scale is increased because of the alignment of j and b at small scales**
- From the induction equation:

$$(\partial_t \mathbf{j})_{\mathrm{withAD}} = (\partial_t \mathbf{j})_{\mathrm{withoutAD}} + \nabla \times [\nabla \times (Re_a^{-1} b^2 \mathbf{j}_\perp)] \quad (1)$$



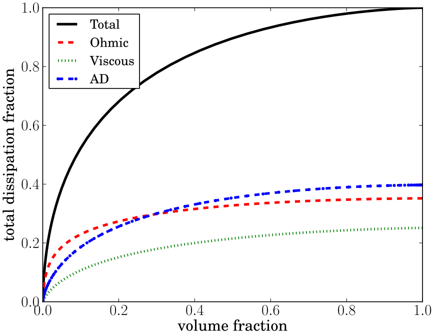
Power spectra of $\mathbf{j} \times \mathbf{b}$.



Compensated dissipation spectrum (AD-OT, $Re_a = 100$).

Incompressible AD MHD - Intermittency

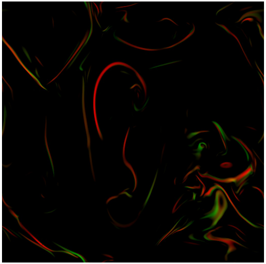
- (Spatial) intermittency: $\varepsilon(\mathbf{x})$ is concentrated on a small subset of the domain.
- Dissipation content vs. volume: 40% of the dissipation in less than 5% of the volume



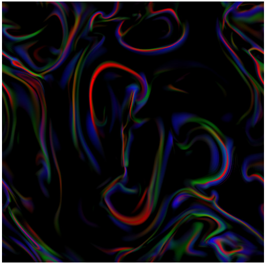
Incompressible AD MHD - Intermittency

The dissipation field - viscous, Ohmic, AD

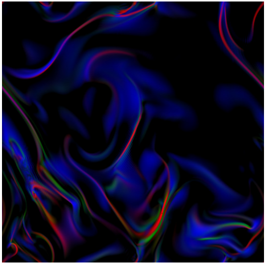
Dissipation concentrated in sheets - AD is more diffuse



MHD - OT, $Re_a = \infty$



AD - OT, $Re_a = 100$



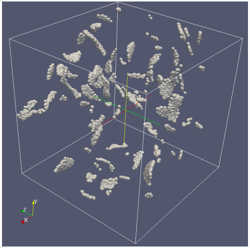
AD - OT, $Re_a = 10$

2D slices of the dissipation field

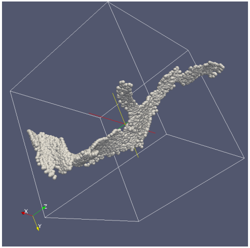
Incompressible AD MHD - Intermittency

Structures of high dissipation - cf. (Uritsky et al. 2010, Zhdankin et al. 2013,2015)

- Post-processing code STRUCT, `http://lerma.obspm.fr/~momferratos/struct.html`
- Defined as connected sets of points having values of the dissipation higher than a given threshold.
- Two algorithms were implemented for their extraction:
  - Recursive, using breadth-first search
  - Iterative



Inertial range structures.

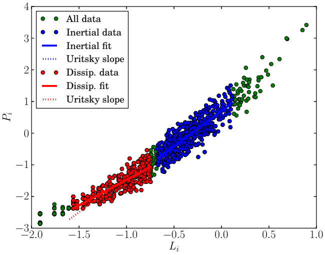


One of the largest structures.

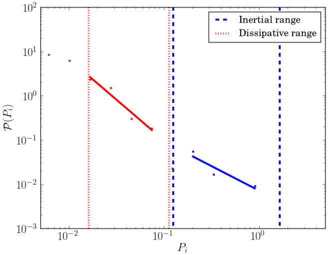
Incompressible AD MHD

Structures of high dissipation - scaling

- Characteristics of structures exhibit power-law scaling $X_i \propto L_i^{D_X}$, L_i: length, X_i: area, volume, volume-integrated dissipation
- The pdf of X_i scales as $\mathcal{P}(X_i) \propto X_i^{-\tau_X}$
- Scaling exponents are different between the inertial and dissipative ranges



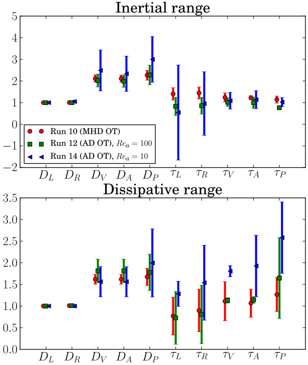
$P_i \propto L_i^{D_P}$



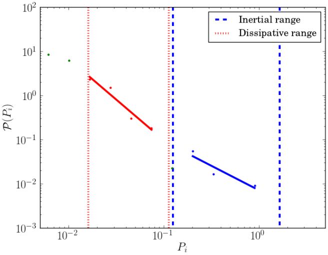
$\mathcal{P}(P_i) \propto P_i^{-\tau_P}$

Weaker structures in the dissipative range.

L_i: length, V_i: volume, A_i: area, P_i: volume-integrated dissipation.



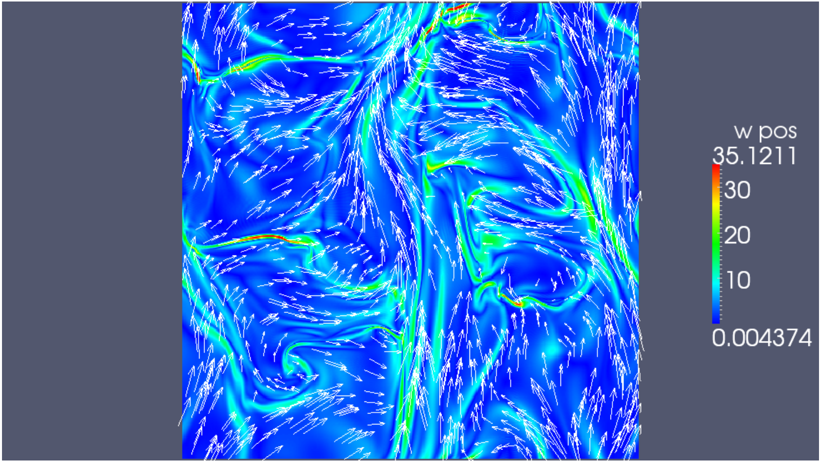
Scaling exponents pure MHD / AD MHD ($Re_a = 100$, $Re_a = 10$)



$$\mathcal{P}(P_i) \propto P_i^{-\tau_P}$$

Vorticity and magnetic field on the plane of the sky

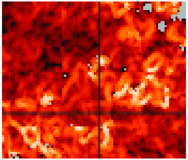
$$\omega_{pos} = \partial_y u_{los} - \partial_x u_{los}$$



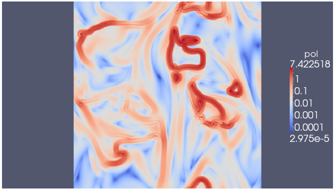
Magnetic field vectors overlaid on contours of vorticity on POS ω_{pos}

Increment of the polarization angle

$$\gamma(x,y,z) = \tan^{-1}(B_z(x,y,z), B_x(x,y,z)^2 + B_y(x,y,z)^2)$$
$$\phi(x,y,z) = \tan^{-1}(B_y(x,y,z), B_x(x,y,z))$$
$$Q(x,y) = \frac{1}{L_d}\int_0^{L_d} \cos(2\phi(x,y,z))\cos^2(\gamma(x,y,z))\, dz$$
$$\delta Q^2(\mathbf{x}, l) = \langle |Q(\mathbf{x}+\mathbf{r}) - Q(\mathbf{x})|^2\rangle_{C_l}$$



Detail from Map of the polarization angle (see Planck Collaboration Int. XIX, 2015, A & A)



Increment of the polarization angle $\delta Q(\mathbf{x}, l)$, $l = 10$ pixels

Conclusions

Results published in Momferratos, G., et al. "Turbulent energy dissipation and intermittency in ambipolar diffusion magnetohydrodynamics." MNRAS 443.1 (2014): 86-101.

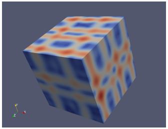
- Study of AD in a 3D turbulent flow using explicit physical dissipation
- **Alignment of j and b increases the AD dissipation scale.**
- The dissipation field is dominated by sheet-like structures, each having a dominant "color", AD is more diffuse
- We measure the statistics of structures of high dissipation
- Comparison with observations: Incompressible simulations reproduce some qualitative features.
- **Perspective**: Study of fractal dimension of structures of high dissipation

Incompressible AD MHD

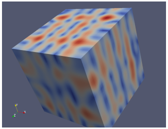
Initial conditions

In all cases, the initial kinetic energy is equal to the initial magnetic energy: $1/2\langle \mathbf{u}^2 \rangle = 1/2\langle \mathbf{b}^2 \rangle$

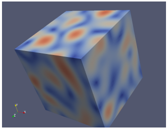
- **Large scale** ($k \in [1,3]$) ABC flow (Arnol'd-Beltrami-Childress)
  - Low cross-helicity $H_c = \langle \mathbf{u} \cdot \mathbf{b} \rangle$
  - Finite magnetic helicity $H_m = \langle \mathbf{a} \cdot \mathbf{b} \rangle, \ \mathbf{a} = \nabla \times \mathbf{b}$
  - **Small scale** random component with energy spectrum
- **Large scale** ($k \in [1,2]$) OT flow (Orszag-Tang vortex)
  - Finite cross-helicity H_c
  - Magnetic helicity H_m zero to machine precision



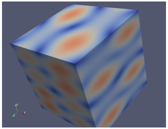
$\mathbf{u}$, ABC



$\mathbf{b}$, ABC



$\mathbf{u}$, OT



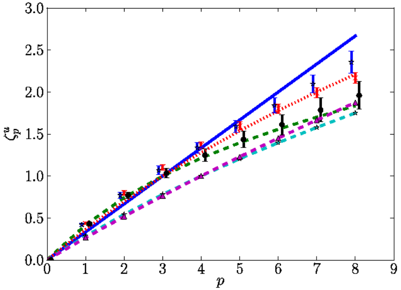
$\mathbf{b}$, OT

Incompressible AD MHD

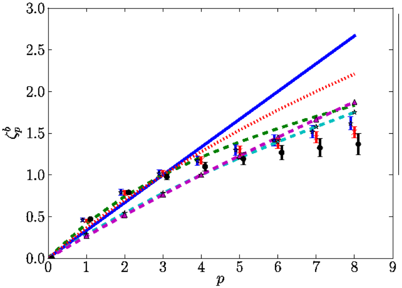
Structure functions

Increased intermittency for the magnetic field

$S_p^u(r) = \langle (\delta u_{\|}(r))^p \rangle \propto r^{\zeta_p^u}$



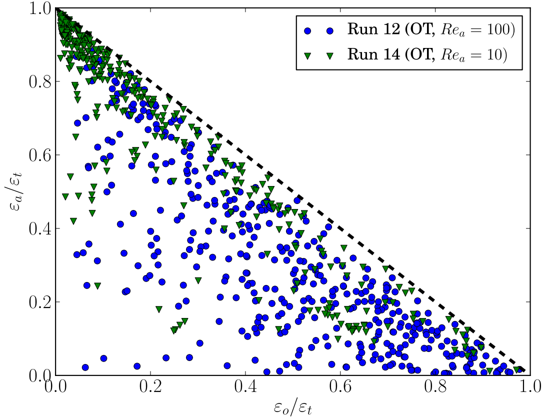
Velocity



Magnetic field

High AD → low viscous dissipation

100% AD - - - -: line of zero viscous dissipation



100% Ohmic

Scatter plots of the ratio $\varepsilon_o/\varepsilon_t$ versus the ratio $\varepsilon_a/\varepsilon_t$ for AD-OT.