The PIC-MHD method and applications

A.Marcowith
Laboratoire Univers et Particules de Montpellier
France

Outlines

- Introduction: Contexts
- Methodology
- Interstellar studies
- Shock studies
- Perspectives
- Conclusions

Contexts

- Transport of energetic particles in turbulent magnetized media
  - Solar corona
  - Interplanetary medium, heliosphere
  - Inter-stellar/-galactic medium
  - Energetic particle sources: shocks, reconnection, shear flows

  …
- Here: 1) focus on interstellar medium and shocks 2) focus on high-energy cosmic rays (E > GeV)

Why MagnetoHydroDynamics ?

- High-energy CRs: Larmor radius $r_L \sim 10^{-3}\,pc\left(\frac{E}{1TeV}\right)\left(\frac{B}{\mu G}\right)^{-1}$

- Resonate with wavelengths $\lambda \sim r_L$ in MHD regime; i.e. long-wavelength regime $\lambda > v_a/\omega_{cp} \sim 6\times10^{-11}$pc $n^{-1/2}$
- Note bene: Not always the case (need to go beyond MHD)
  - MeV particle transport (Jean+09)
  - Thermal-non-thermal transition as in the shock injection process (Levinson'96)
  - Relativistic shock turbulence (Pelletier+09, Plotnikov+13)

Methodology I: Particle-in-cell

- Solving the Lorentz equation:

$$\frac{d\mathbf{p}}{dt} = q\delta\mathbf{E} + q\left(\mathbf{v} \times (\mathbf{B} + \delta\mathbf{B})\right)$$

- B background (largest scale) magnetic field + (δE,δB) perturbed EM components.
- The EM field is known on a grid => interpolation at the particle position.
- The Lorentz Eq. has to be integrated => integration schemes.

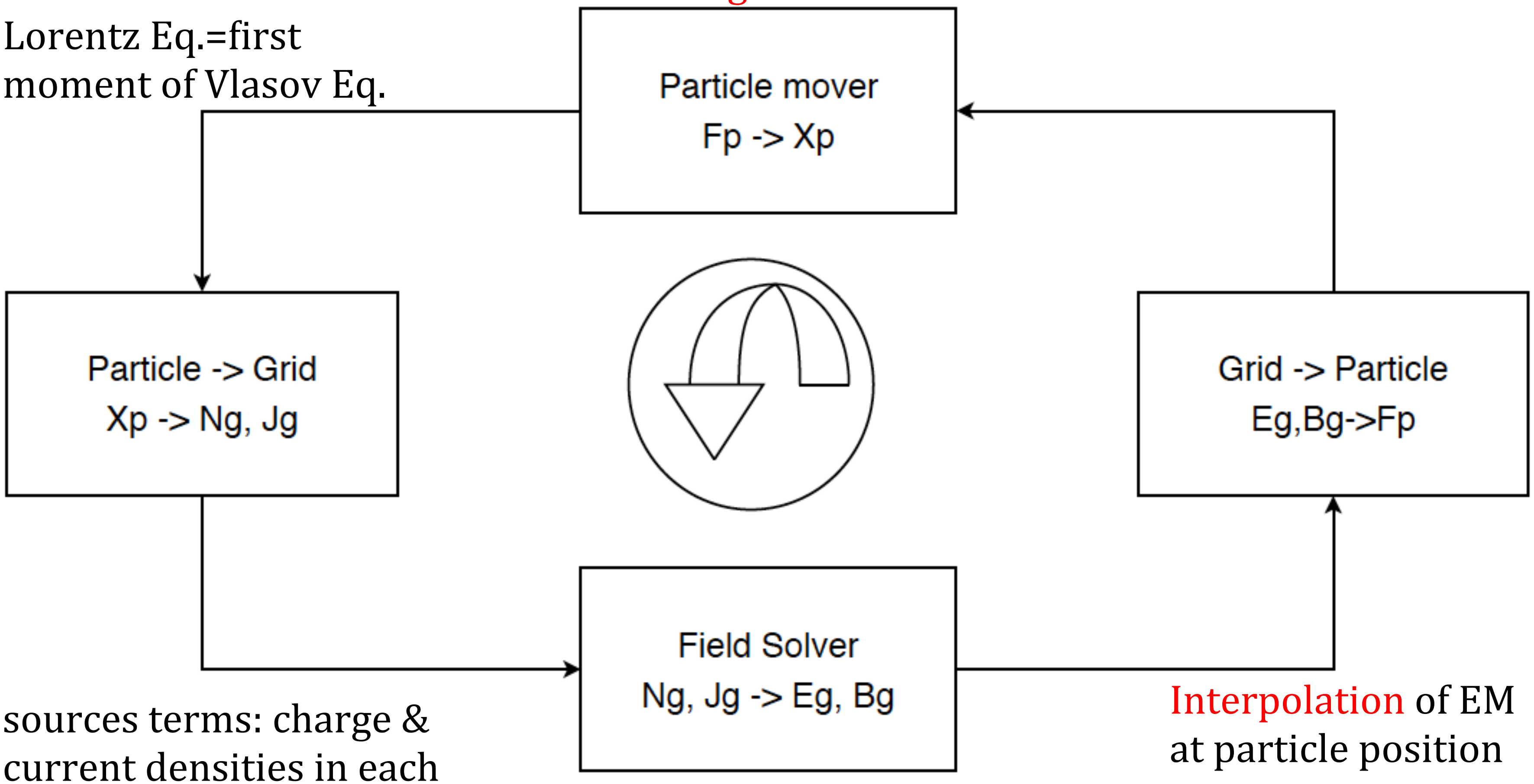
(see Lapenta: https://perswww.kuleuven.be/~u0052182/pic/book.pdf; Birdsall & Langdon'04)

Particle-in-cell code integration sketch

PIC method a particle = super-particle = N_p particle per grid cell

Lorentz Eq.=first moment of Vlasov Eq.

Integration



sources terms: charge & current densities in each cell (interpolation)

EM on the grid

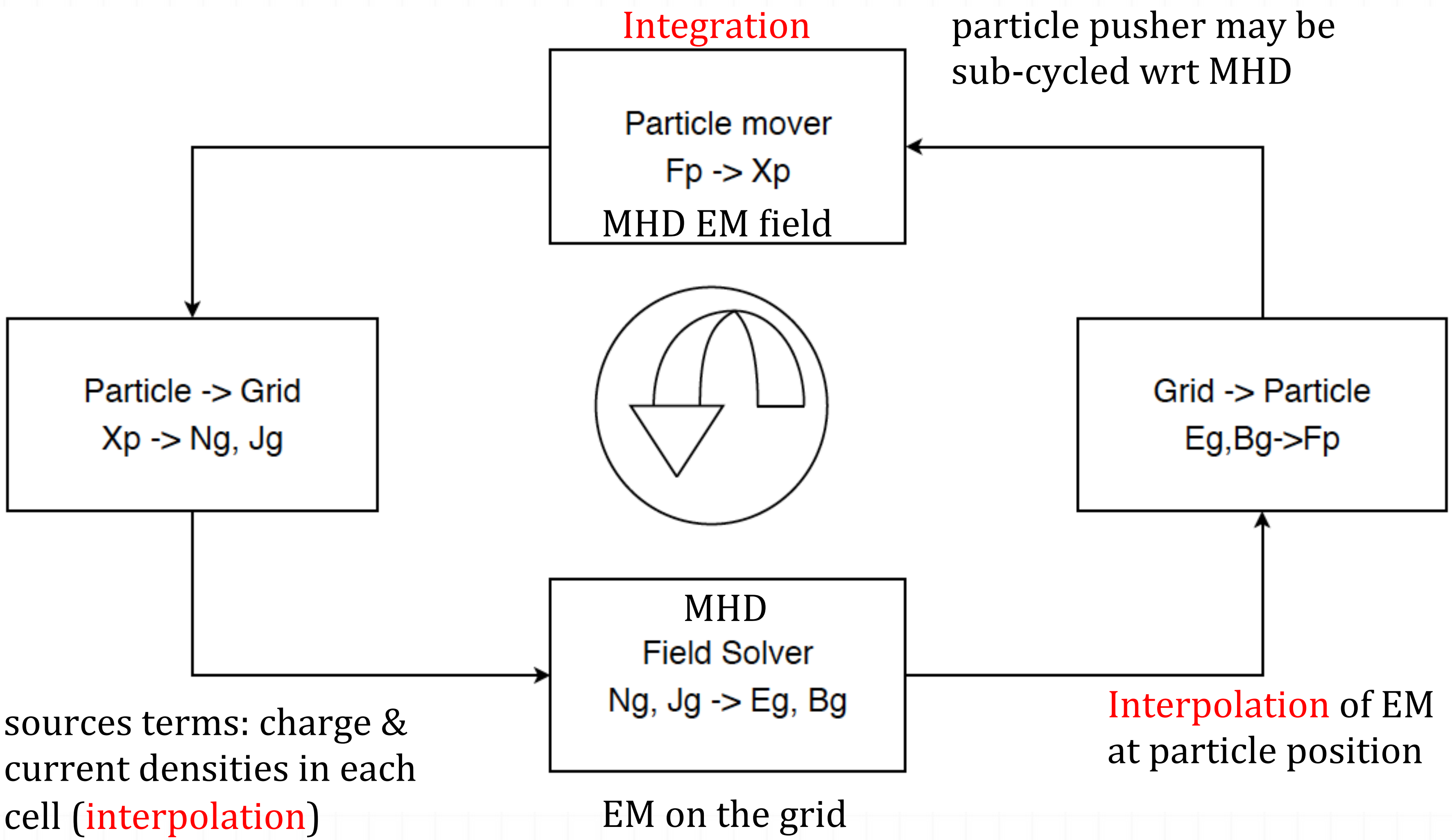
Interpolation of EM at particle position

https://perswww.kuleuven.be/~u0052182/pic/book.pdf

Methodology II: PIC-MHD

- The EM field is calculated from a MHD code.
- Until now MHD = *one fluid MHD* = electron+proton, PIC=energetic particles only.
- Different from hybrid methods (e.g. Gargaté'07), or pure PIC methods (Lorentz+Maxwell Eqs system).

Particle-in-cell –MHD code integration sketch



https://perswww.kuleuven.be/~u0052182/pic/book.pdf

An alternative method (useful in astrophysics): multi-fluid approach

Adding a (or several) CR fluid component:

(e.g. Dubois & Commerçon'15, Hanasz & Lesch'03)

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \boldsymbol{u}) = 0\,, \qquad (1)$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla.\left(\rho \boldsymbol{u}\boldsymbol{u} + p_{\rm tot} - \frac{\boldsymbol{B}\boldsymbol{B}}{4\pi}\right) = 0\,, \qquad (2)$$

$$\frac{\partial e}{\partial t} + \nabla.\left((e + p_{\rm tot})\boldsymbol{u} - \frac{\boldsymbol{B}(\boldsymbol{B}.\boldsymbol{u})}{4\pi}\right) = -\nabla.\boldsymbol{F}_{\rm cond} - \nabla.\boldsymbol{F}_{\rm CR}\,, \qquad (3)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) = 0\,, \qquad (4)$$

$$\frac{\partial e_{\rm E}}{\partial t} + \nabla.(e_{\rm E}\boldsymbol{u}) = -p_{\rm E}\nabla.\boldsymbol{u} - \nabla.\boldsymbol{F}_{\rm cond} + \mathcal{H}_{\rm EI} \qquad (5)$$

$$\frac{\partial e_{\rm cr}}{\partial t} + \nabla.(e_{\rm cr}\boldsymbol{u}) = -p_{\rm cr}\nabla.\boldsymbol{u} - \nabla.\boldsymbol{F}_{\rm CR}\,, \qquad (6)$$

$$p_{tot} = (\Gamma - 1)e_{th} + (\Gamma_{CR} - 1)e_{CR} + B^2/8\pi$$

Difficult task: Div (F_{CR})
⇒ implicit schemes (see Dubois & Commerçon'15)

+ Handling back-reaction, can use several CR populations.
- Specify Γ_{CR}, κ (averaged diffusion coefficient)

=> Novak, Hanasz posters

Application I: interstellar studies

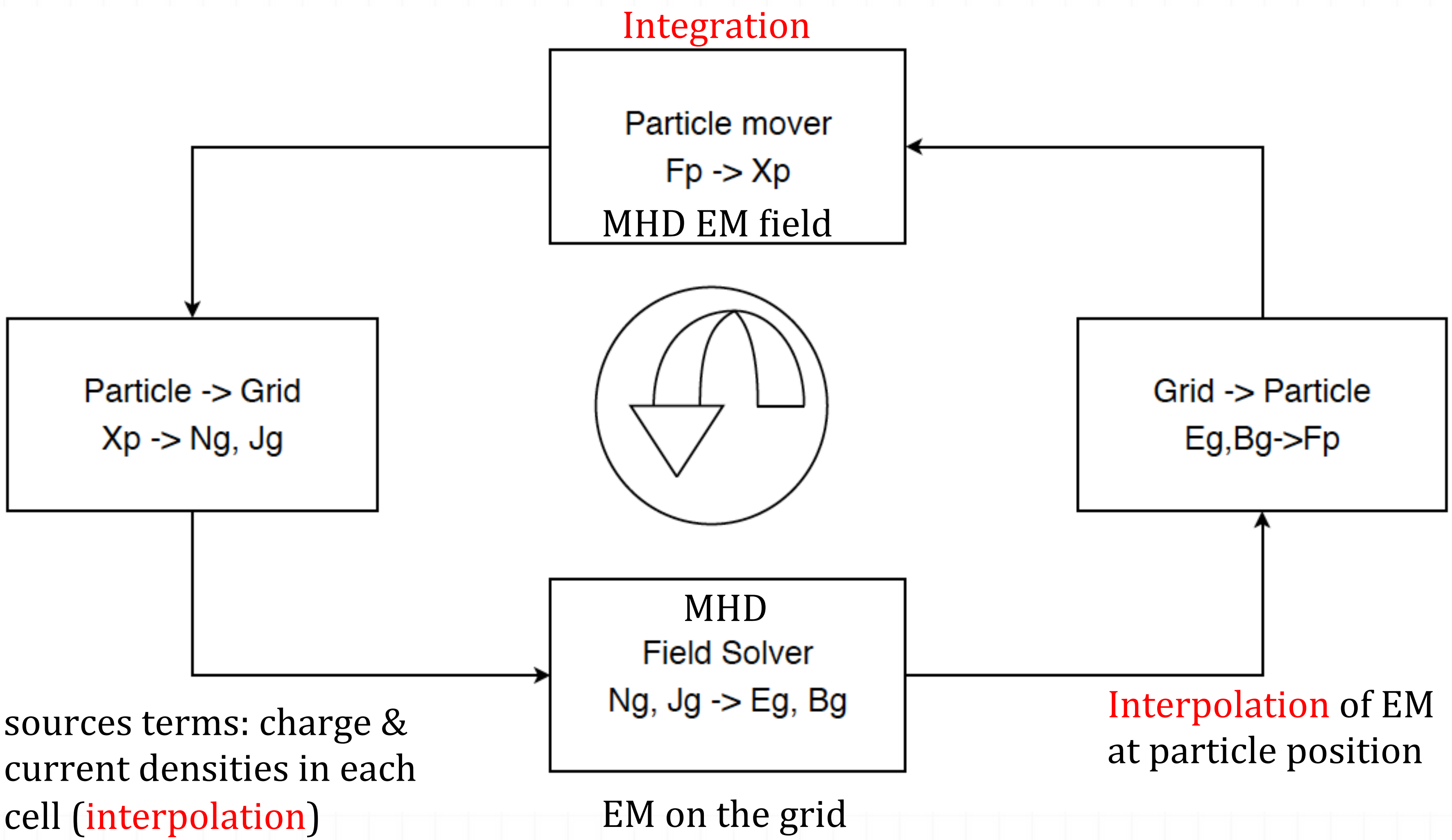
- Cosmic ray transport in MHD turbulence in the interstellar medium.

- Important aspect: $V_a/c << 1$
  - If we consider relativistic particles moving in ISM, hence
  - Magnetostatic limit can be used (neglect δE)
  - In case of propagation of TeV-PeV particles hence ρ_{CR}, J_{CR} can be neglected in the MHD source terms: Test-particle limit.

  => Seta poster

- This does not mean that stochastic acceleration is not intersting or that TeV-PeV CR cannot back-react over the magnetized fluid (around/in CR sources)

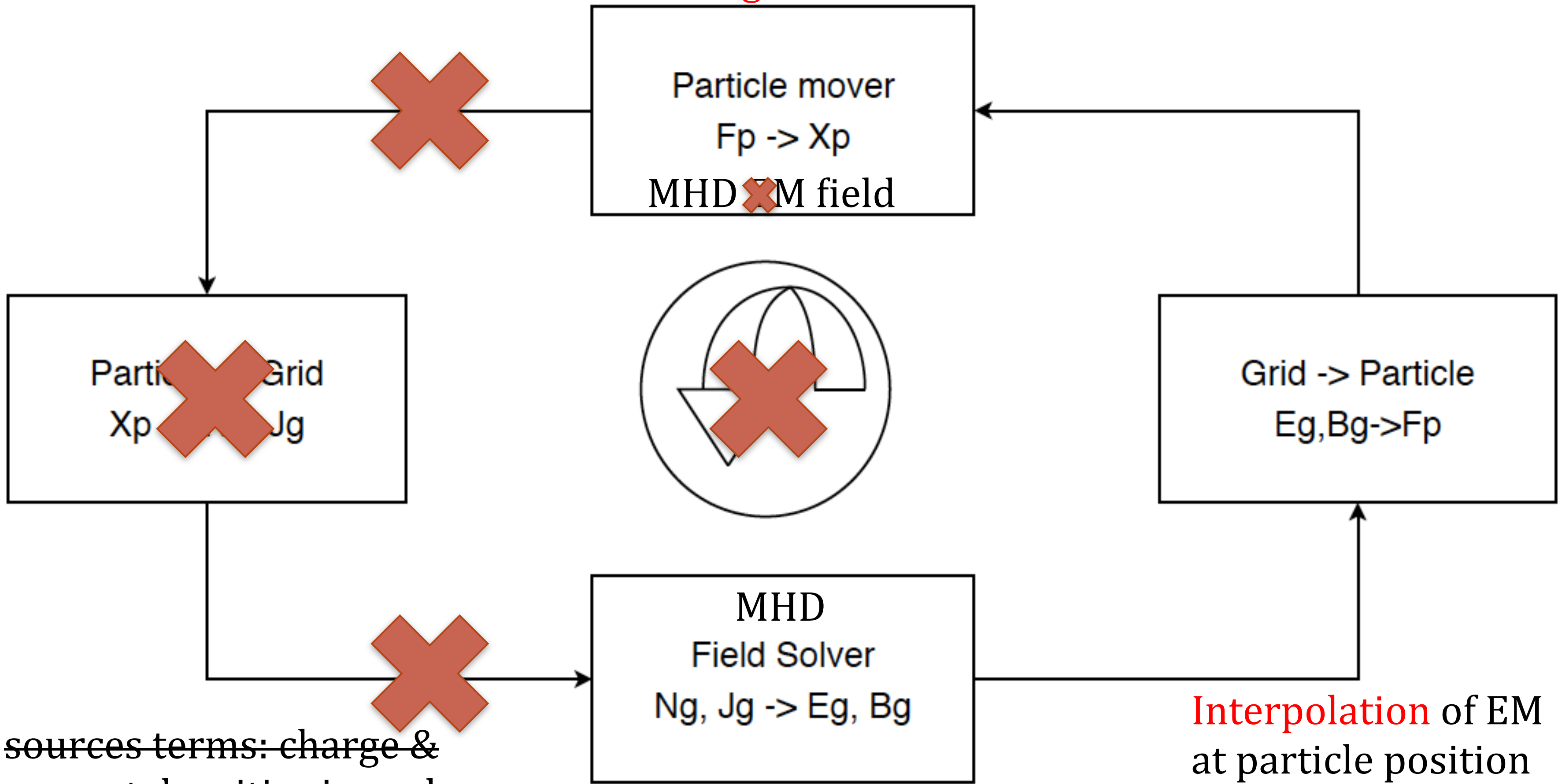
Particle-in-cell –MHD code integration sketch



https://perswww.kuleuven.be/~u0052182/pic/book.pdf

Particle-in-cell –MHD code integration sketch: magnetostatic test-particle calcualtions

Integration



~~sources terms: charge & current densities in each cell (interpolation)~~

Interpolation of EM at particle position

EM on the grid

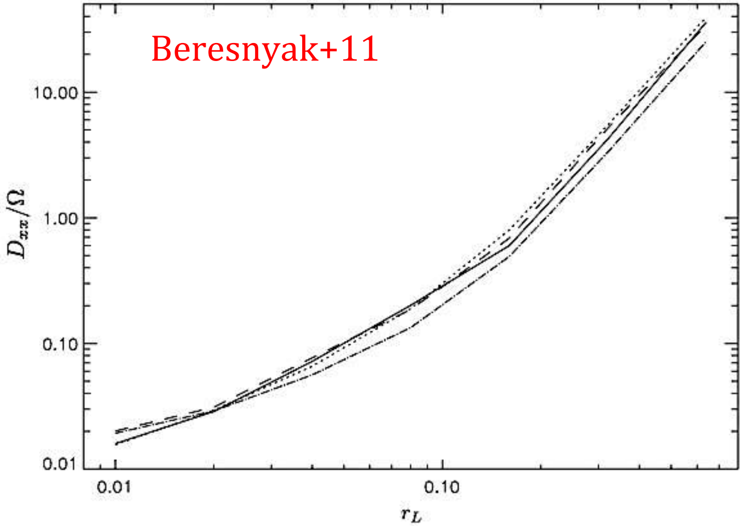
https://perswww.kuleuven.be/~u0052182/pic/book.pdf

Large scale injected turbulence

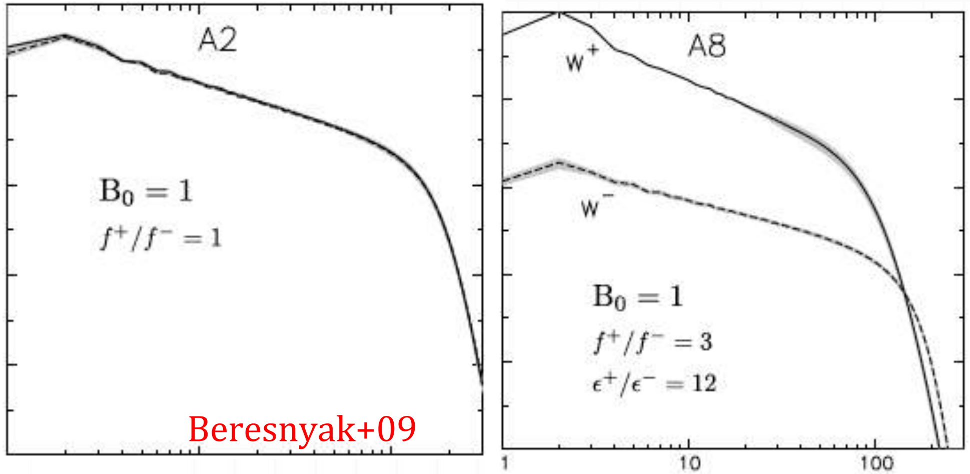
- <u>Set-up</u>:
  - 3D simulations periodic boxes
  - MHD snapshots ⬄ magnetic realizations
  - Cosmic Ray mean free paths: integrate trajectories and average over the magnetizations.
- <u>Forcing</u>:
  - Forcing the velocity field => F in the source term of Euler Eqs.
  - Usually incompressible forcing: div.F=0. : In the ISM ⬄ shear flows.

Incompressible MHD turbulence

parallel Dr_L *versus* r_L



Large-scale-injected turbulence



Solid: balanced, dot-dashed: strongly imbalanced trans-Alfvénic case

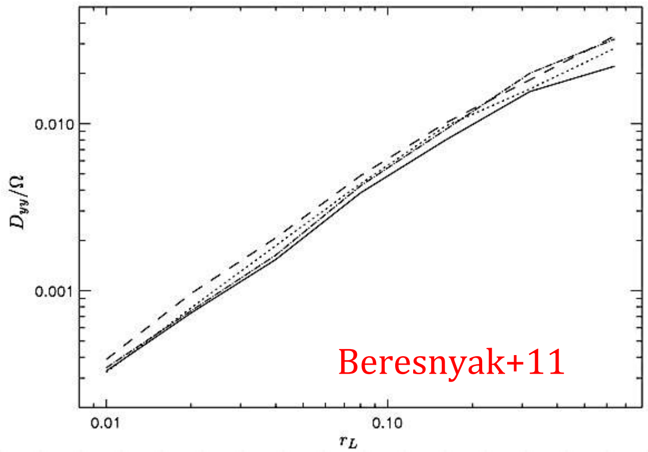
D weakly dependent on r_L

Weak effect of imbalancing

Perpendicular transport by field line wandering

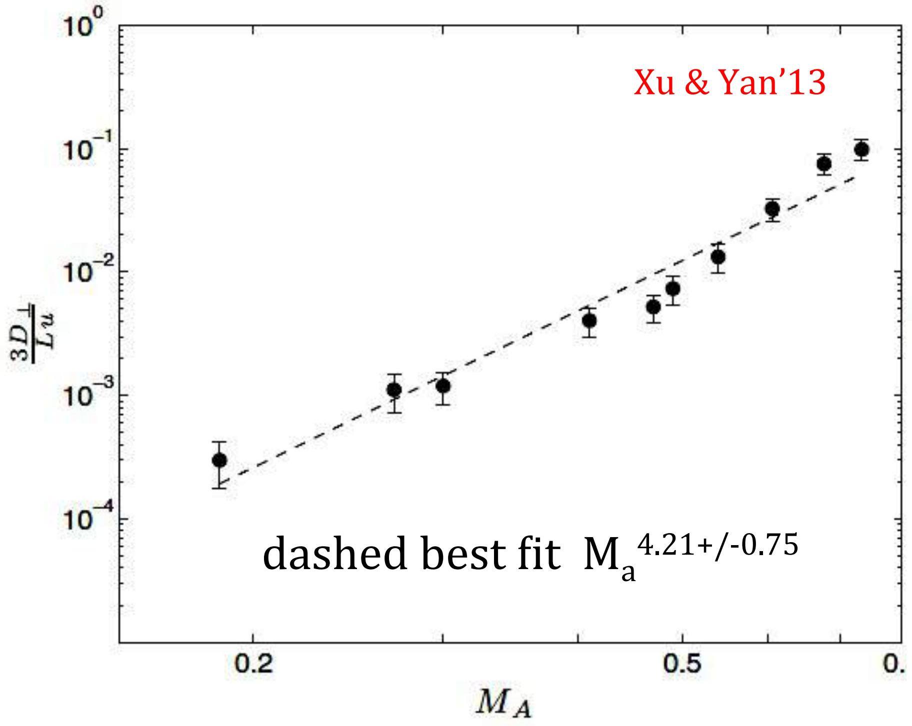
⇔ Transport controlled by large scales

perpendicular Dr_L *versus* r_L

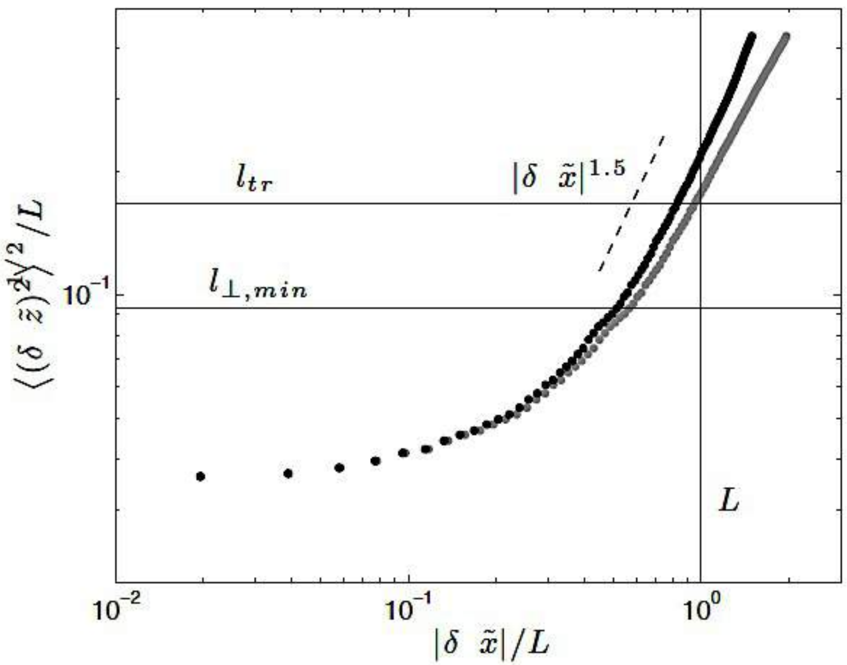


Compressible MHD turbulence

perpendicular mean free path *versus* M_a



rms distance between particles *versus* parallel distance along MF



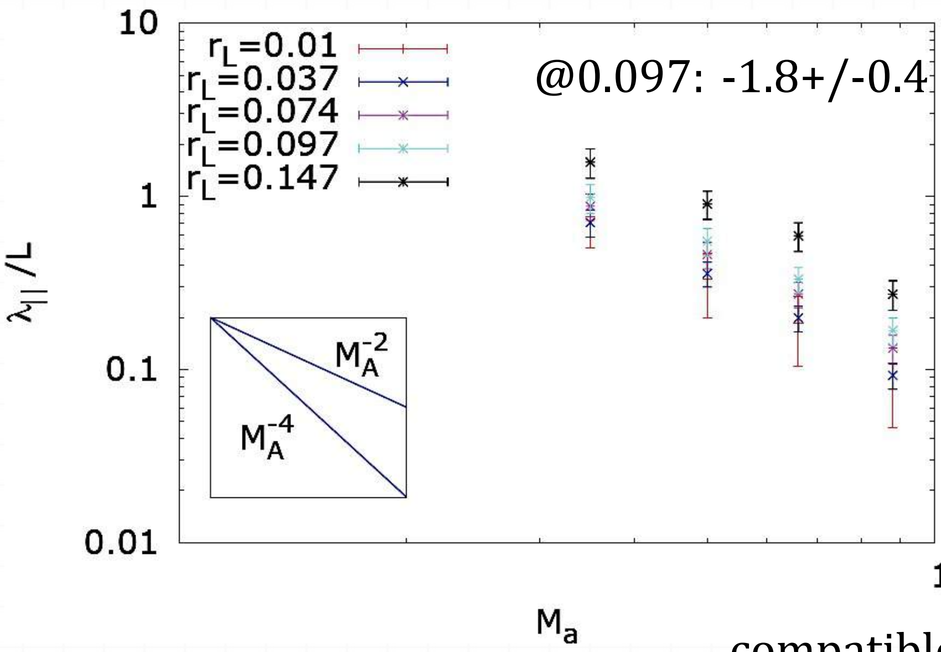
Consistent with M_a^4 scaling expected in sub-Alfvénic turbulence with parallel mean free path > L_{coh} (Yan & Lazarian'08)
expected: $M_a^4\ L_{coh}\ v/3$

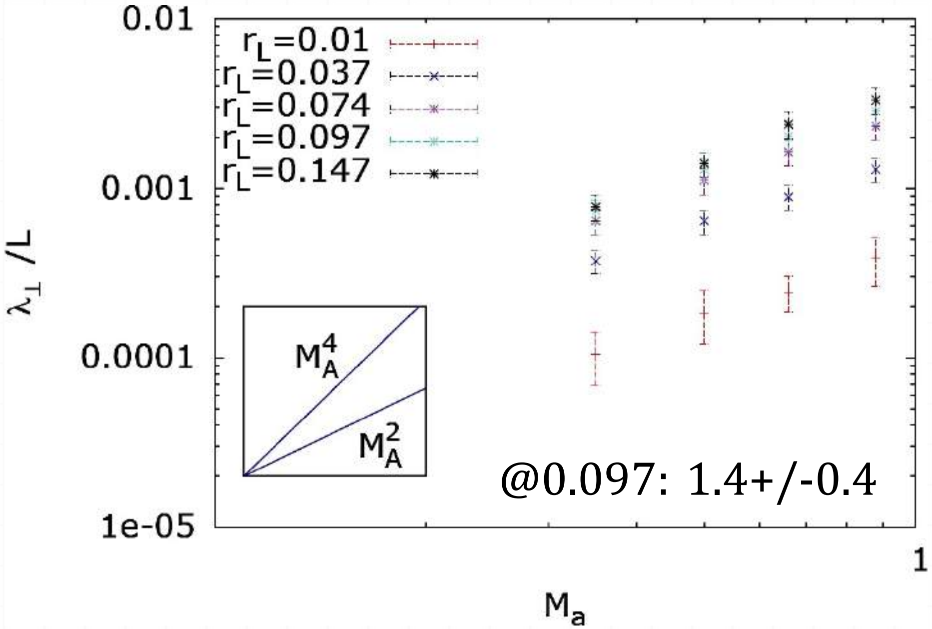
perpendicular transport over scales < L_{coh} at M_a=0.4

super-diffusive transport

Compressible MHD turbulence: effect of forcing geometry

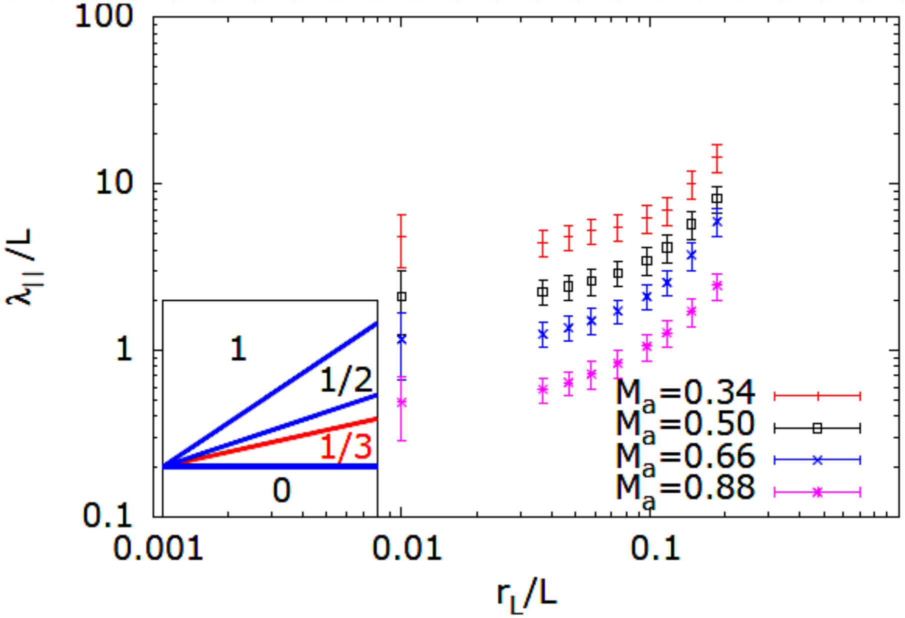
- Ornstein-Uhlenbeck forcing (Federrath+08) with both incompressible (div.F=0) and compressible (curl F=0) geometries (Cohet & AM'15).
- In the ISM we can also have compressible mode injection ⇔ supernova remnants, massive star winds.





compatible with QLT predictions

rigidity dependences



Parrallel MFP indices

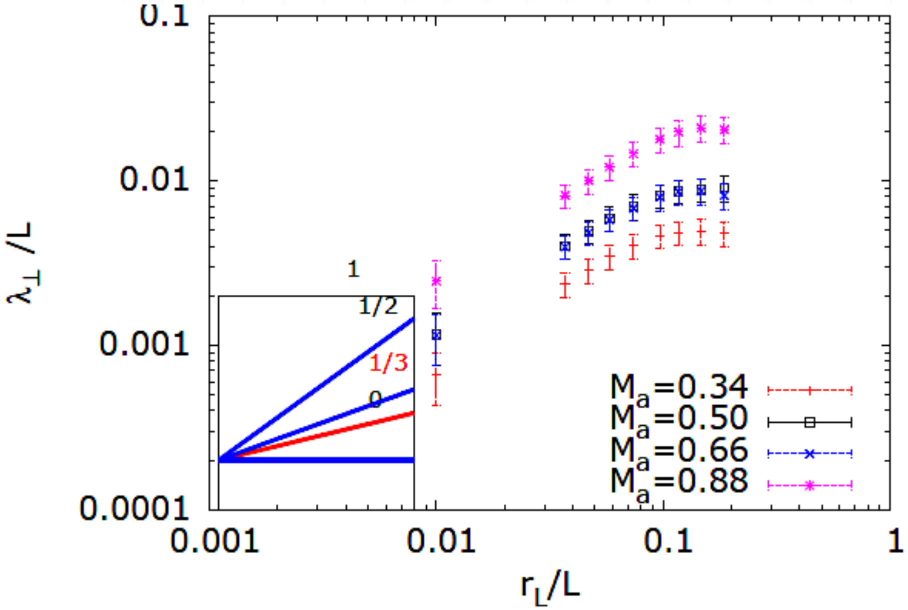
M_a	$\alpha(0.03-0.1)\pm\Delta\alpha$	$\alpha(0.01-0.1)\pm\Delta\alpha$
0.35	0.34 ± 0.39	0.16 ± 0.20
0.50	0.44 ± 0.37	0.29 ± 0.21
0.66	0.54 ± 0.35	0.37 ± 0.19
0.89	0.62 ± 0.37	0.44 ± 0.18

consistent with 1/2 or 1/3

Perpendicular MFP indices

M_a	$\alpha(0.03-0.1)\pm\Delta\alpha$	$\alpha(0.01-0.1)\pm\Delta\alpha$
0.34	0.70 ± 0.34	0.86 ± 0.19
0.50	0.73 ± 0.34	0.86 ± 0.21
0.66	0.85 ± 0.35	0.94 ± 0.18
0.89	0.82 ± 0.35	0.89 ± 0.22

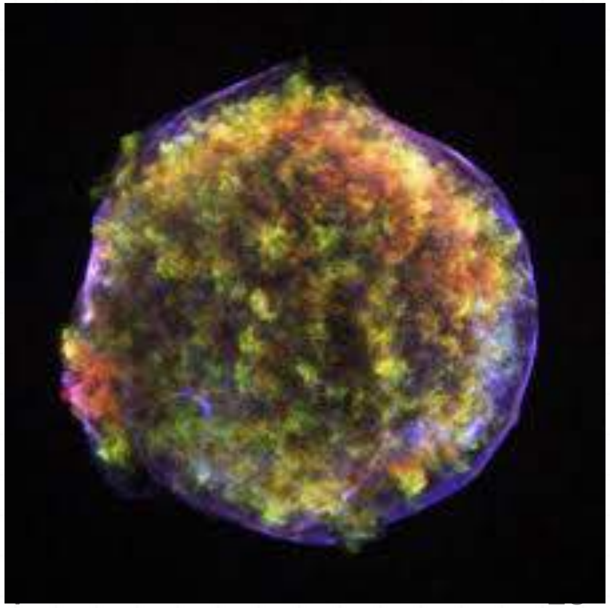
consistent with 1/2



Application II: Astrophysical shocks

- PIC-MHD are coupled => source terms in MHD Eq.
- Diffusive shock acceleration and magnetic field amplification in supernova remnant (SNR) shocks.
  - Magnetic field is turbulent in young SNR.
  - Field amplitude can be two orders of magnitude above standard ISM values.

Tycho SNR
4-6 keV X-rays: blue ⇔ synchrotron radiation by TeV electrons



PIC-MHD shock studies

- Precursor studies:
  - CR propagation in snapshots Reville+08
  - Full coupling Reville &Bell'11
- Full shock structure studies:
  - Non-resonant streaming instability and CR acceleration Bai+15

Sources terms in MHD Eq.

- CR: charge density n_{CR} and current $J_{CR}=n_{CR}\mathbf{u}_{CR}$: calculated from the sum of contribution of each CR.
- The main effect of CR => CR-Hall term in the Ohm law; namely:

$$E=-\frac{u_g}{c}\times B-\frac{n_{CR}}{|n_e|}\frac{\left(u_{CR}-u_g\right)}{c}\times B$$

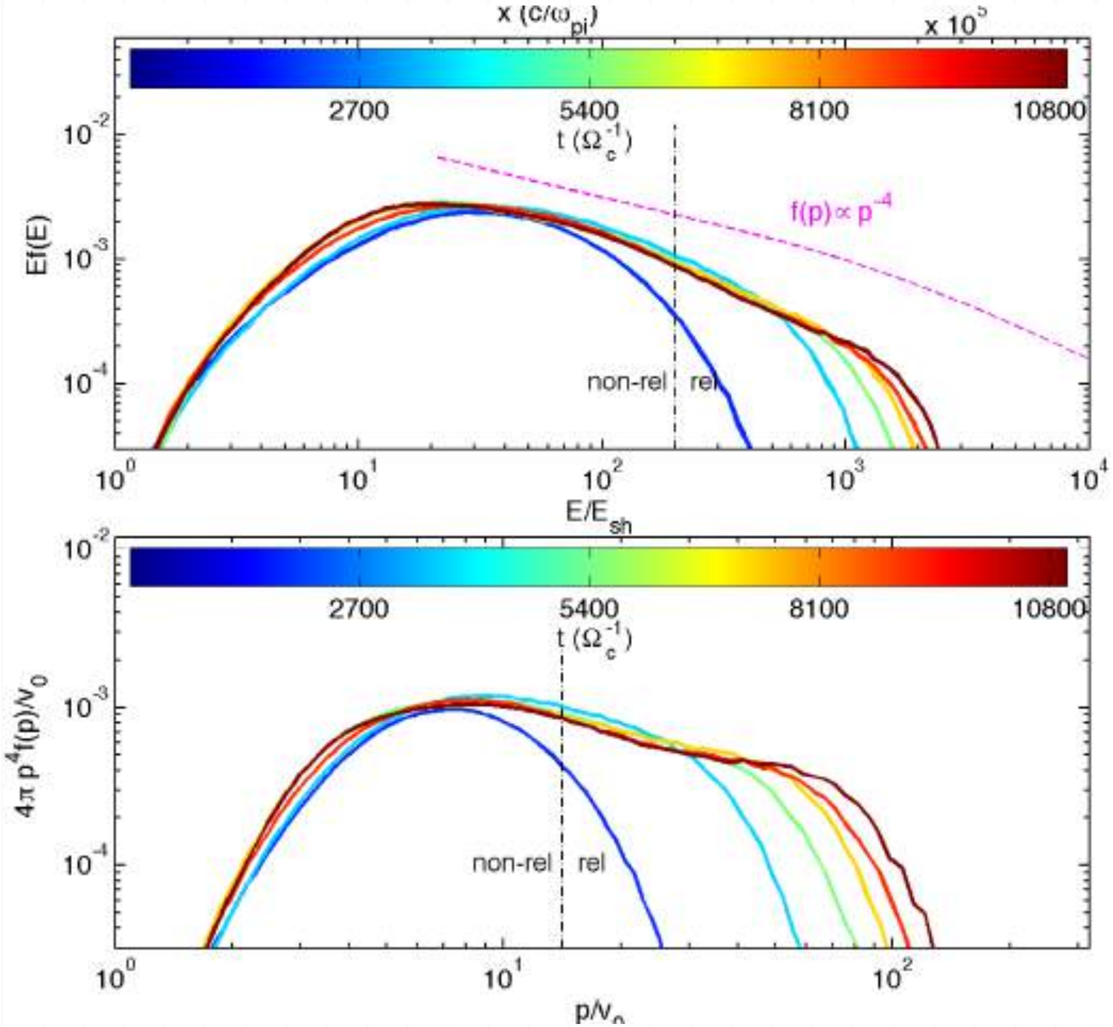
In highly super-Alfvenic shocks $u_{CR}\sim u_{sh} >> V_a \sim u_g$ it compensates $n_{cr} << |n_e|$

- The force induced by CR over the fluid:

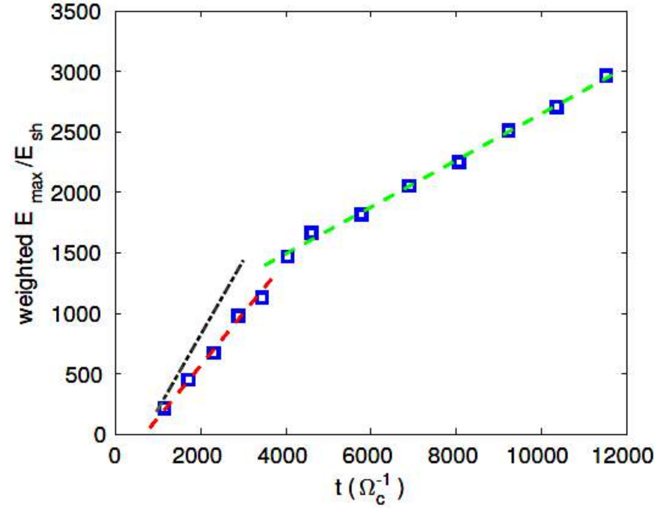
$$F_{CR}=(1-R)\left(n_{CR}E_{ind}+J_{CR}/c\times B\right);R=n_{CR}/|n_e|<<1$$

2D simulations

transition to relativistic regime



E_{max} (t)



Perspectives

- Numerics: some lands to clear:
  - PIC-MHD on AMR grids.
  - Relativistic PIC-MHD.
- Physics: (multi)thesis subjects:
  - ISM studies:
    - MHD modes impact over CR transport.
    - Self-generated wave contribution.
    - CR back-reaction over ISM.
  - Shock studies:
    - Parametric survey (magnetization, obliquity, velocity regime, see e.g. L.Sironi this session in the case of relativistic shocks).
    - CR escape problem

Conclusions

- PIC-MHD methods:
  - Investigate scales related to mildly-relativistic to relativistic cosmic rays.
  - Interstellar medium studies
    - Parallel mean free path: effect of MHD modes (forcing studies)
    - Perpendicular mean free path: test of the effect of field line wandering and analytic turbulence models
  - Shock studies:
    - development of non-resonant instability in fast supernova remnant shocks.
    - injection into the relativistic domain and Fermi acceleration seems to be verified.

Back-up

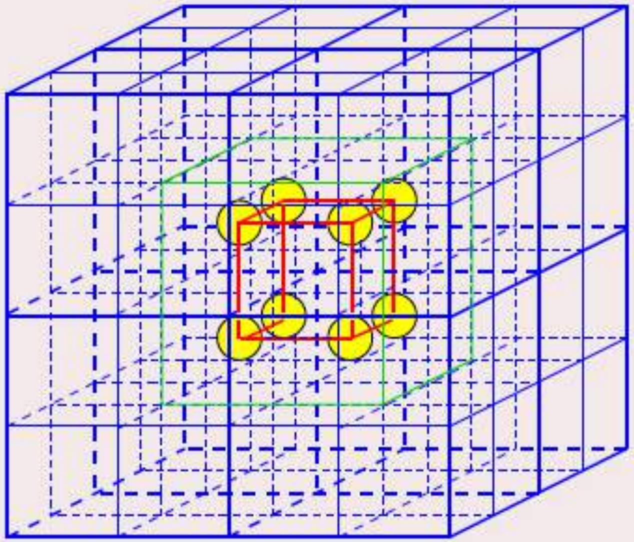
- Examples of Lorentz Eq. integration schemes.
- Examples of EM field interpolation schemes.

Integration schemes

- Main schemes:
  - Leap-frog (second order scheme).
  - Runge-Kutta 5th order.
  - Bulirsch-Stoer method.

Field interpolation schemes

$$\delta = x_p - x_c$$

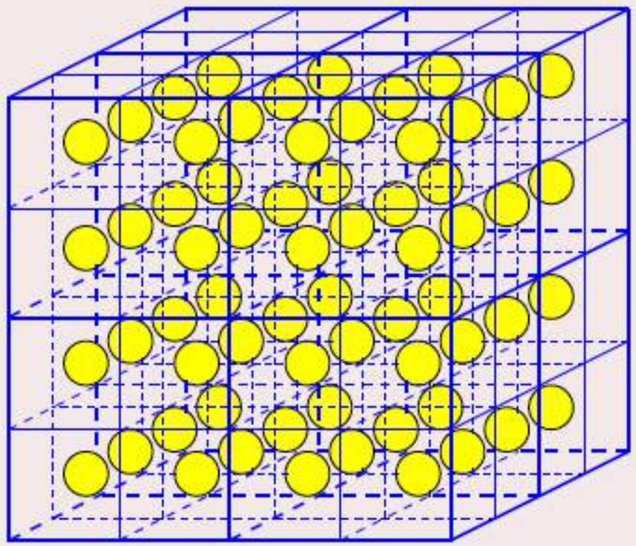


- Cloud-In-Cell (CIC) *(Teyssier, 2002)*

$$S^1(\delta) = \begin{cases} 1 - |\delta| & \texttt{if } |\delta| < 1 \\ 0 & \texttt{otherwise} \end{cases}$$

- Piecwise Cubic Spline (PCS) *(Haugbolle, 2012)*

$$S^3(\delta) = \frac{1}{6}\begin{cases} 4 - 6\delta^2 + 3|\delta|^3 & \texttt{if } |\delta| < 1 \\ (2 - |\delta|)^3 & \texttt{if } |\delta| \in [1;2[\\ 0 & \texttt{otherwise} \end{cases}$$



Non-resonant instability

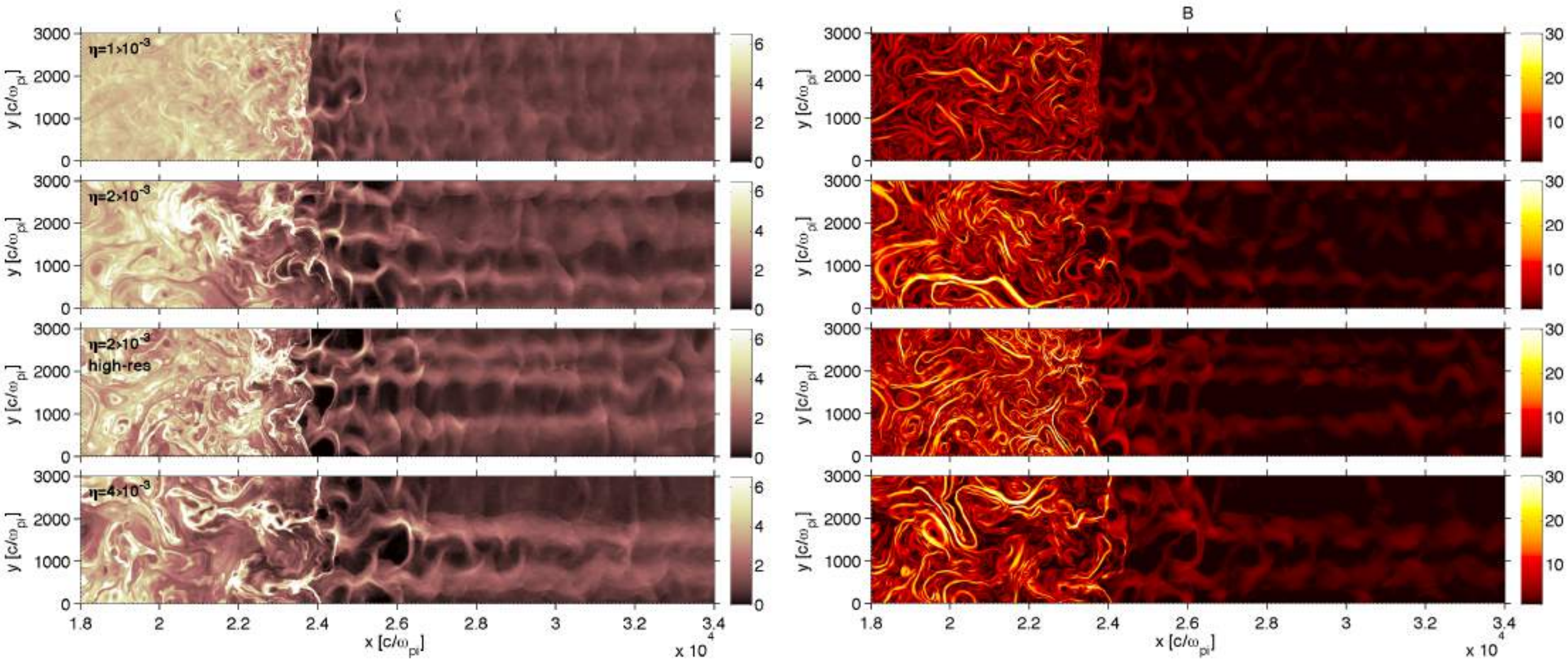
- Instability triggered by the super-Alfvénic streaming of CR in the background (ISM) medium (Bell'04, '05)
  - Generates modes $\lambda << r_L$ (Larmor radius of triggering particles)
  - Growth rate + unstable wave-number:

- Added-value PIC-MHD (CR-Hall term effect) system (Bai+15)
  - Reduced growth rate and wave number.

effect of injection

density

magnetic field



- Filaments develop up-stream ⇔ non-linear stage of the non-resonant instability