Magnetic Plasma Turbulence: Theory meets Observations

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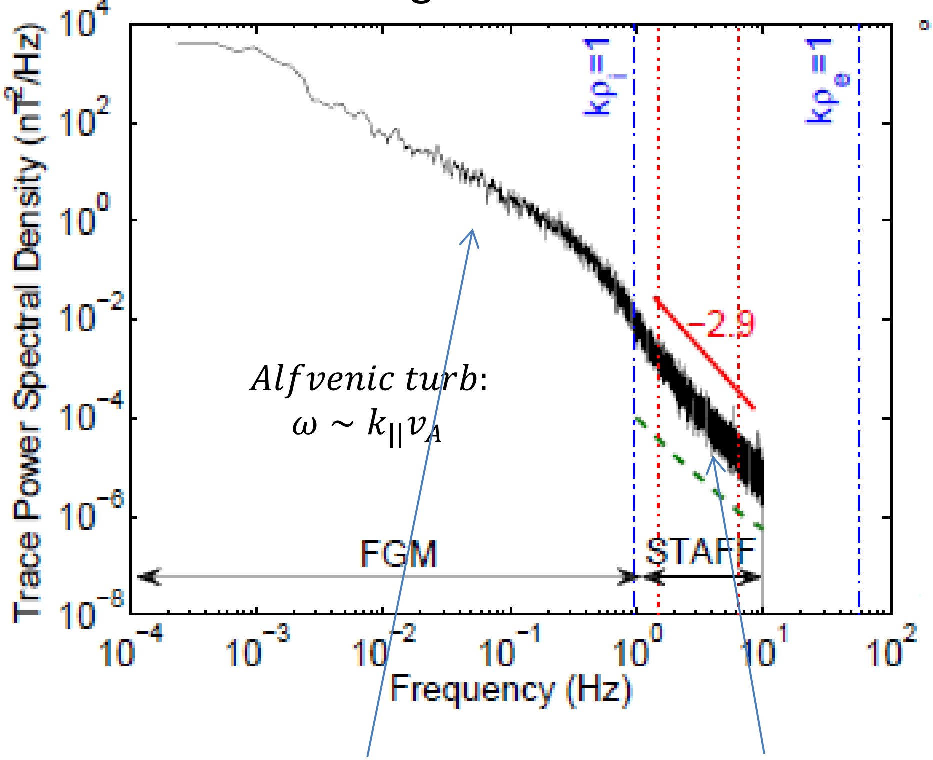
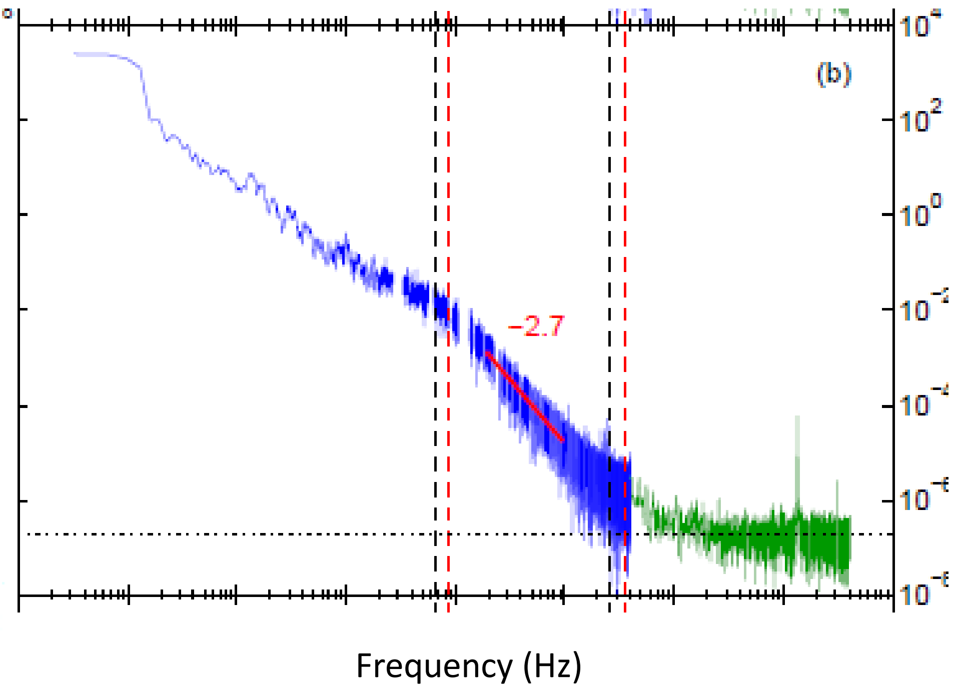
Fluctuations in the solar wind

Magnetic field



$Alfvenic\ turb$:
$\omega \sim k_{||} v_A$

Density



Frequency (Hz)

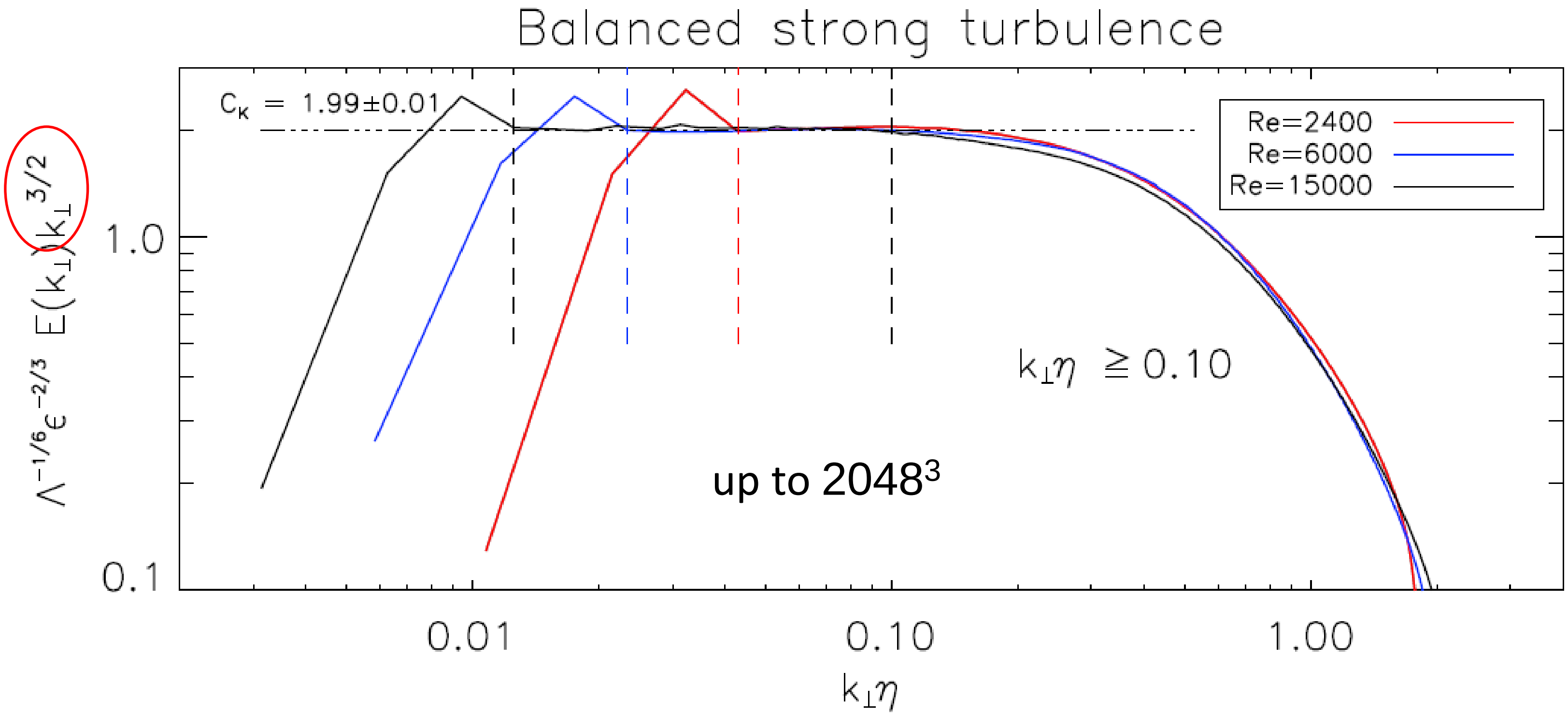
MHD-scale turbulence

Kinetic-scale turbulence

Chen, et al (2010)

Chen, et al (2012);
Safrankova et al (2013)
Mozer & Chen (2013) for electric field

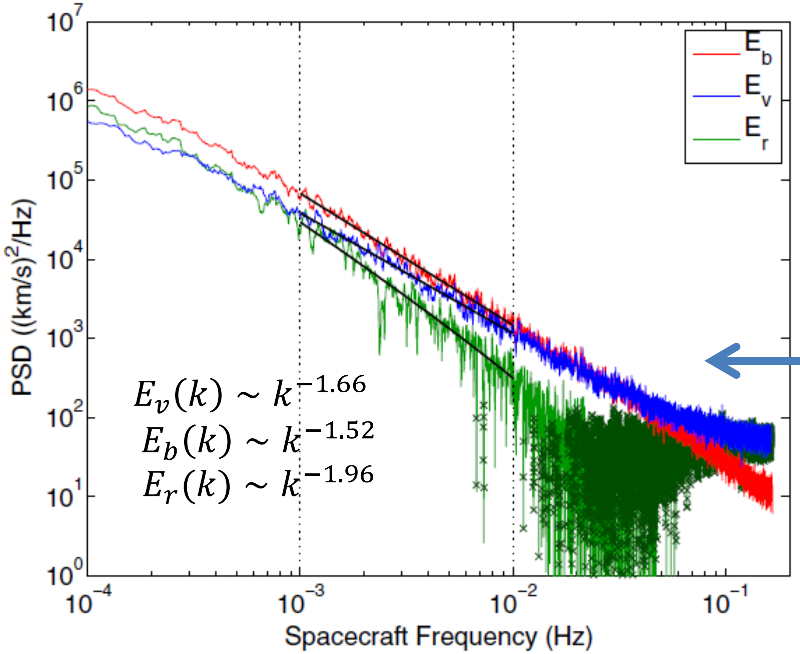
MHD turbulence in numerical simulations



Computational resources: DoE 2010 INCITE,
Machine: Intrepid, IBM BG/P at Argonne Leadership Computing Facility

Perez et al, Phys Rev X (2012), ApJL(2014)

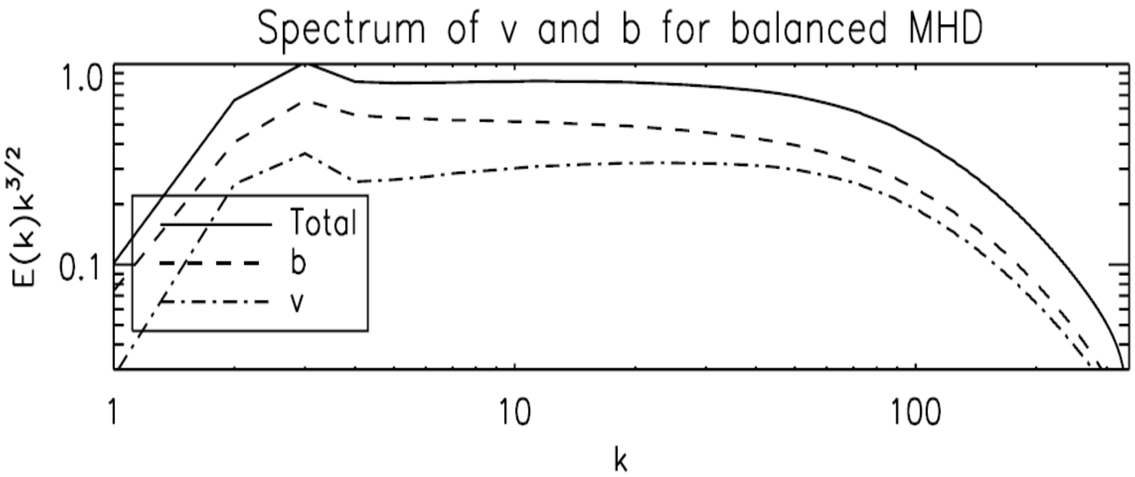
Energy and residual energy



Residual energy $E_r(k)=E_b(k)-E_v(k)$:
Excess of magnetc energy over kinetic

Solar wind (Chen et al 2013)

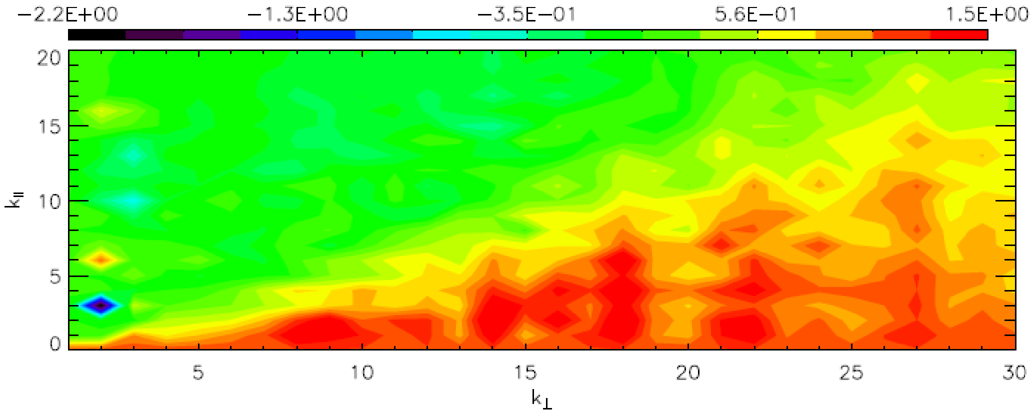
Numerics
(SB, Perez, Borovsky, Podesta 2011)



Residual energy is spontaneously generated,
not supplied by the forcing

Residual energy in weak MHD turbulence has nontrivial structure

$$e^r(k) = Re\langle z^+(k) \cdot z^-(k) \rangle \propto -\epsilon^2 k_\perp^{-2} \Delta(k_{||})$$

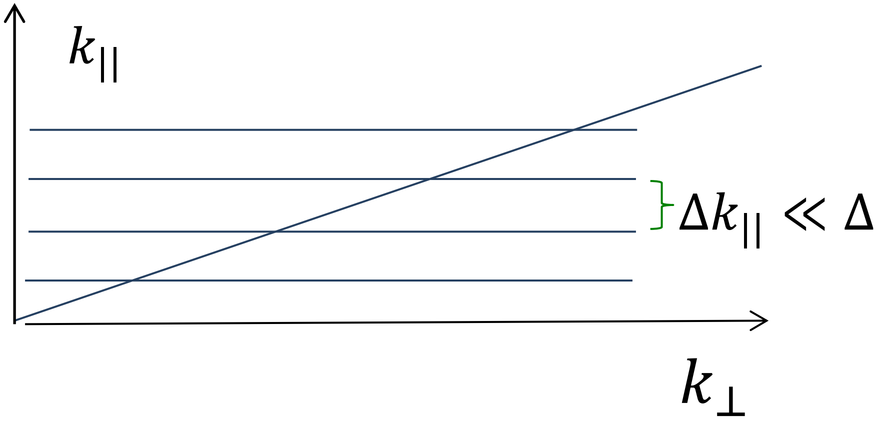


$\Delta(k_{||})$ is concentrated at

$$k_{||} < C\epsilon^2 k_\perp$$

Wang et al (2011)

"condensate"

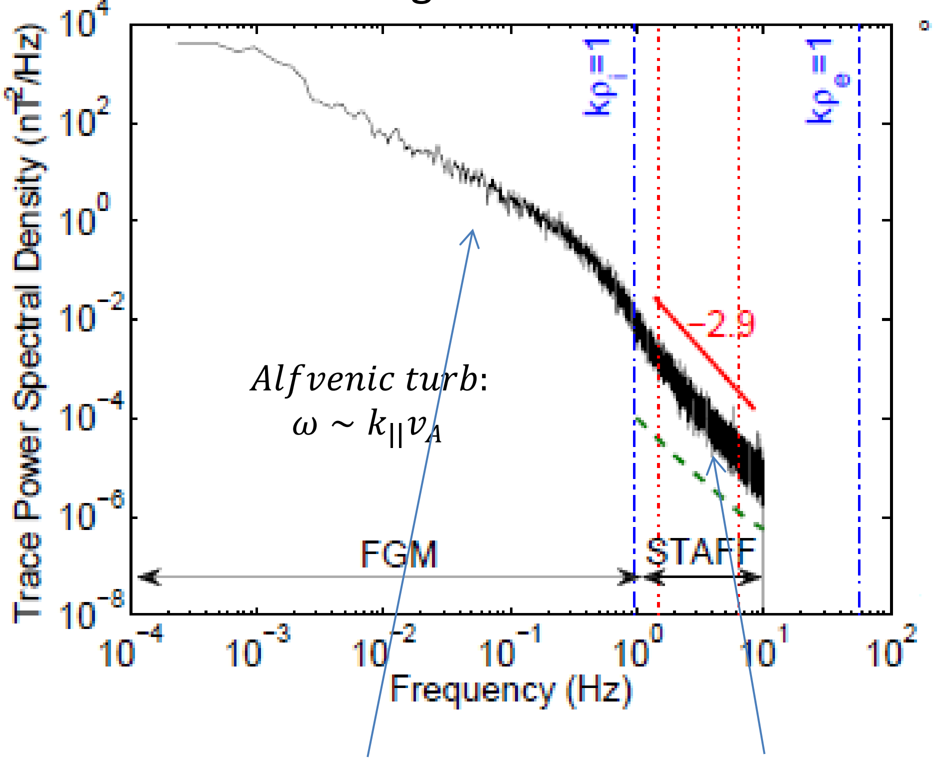


Structure in k-space must be resolved in numerical Simulations.

Strong turbulence? Solar wind?

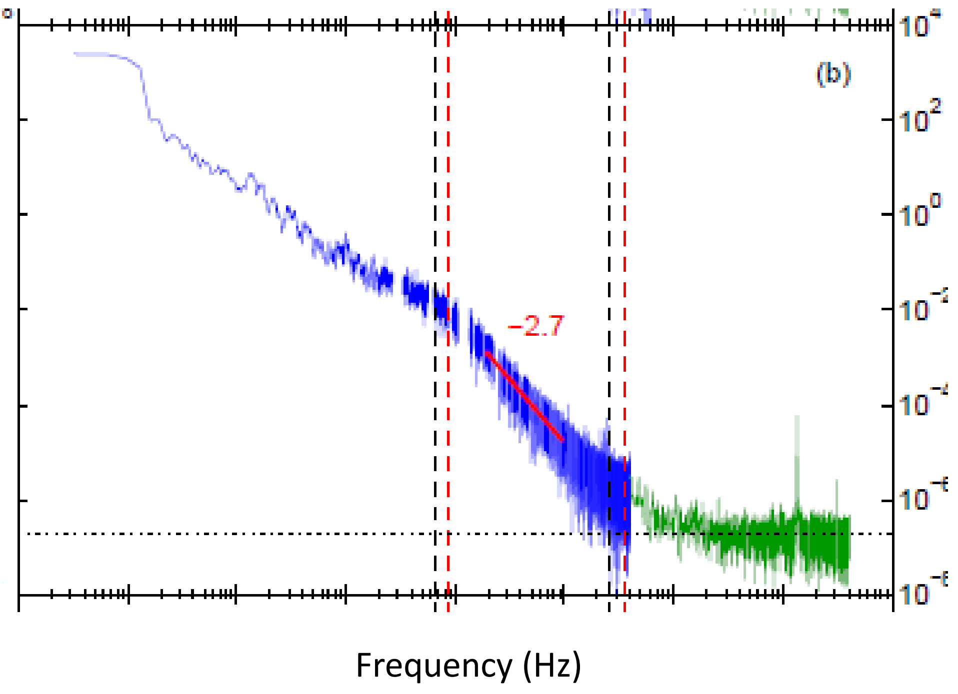
Fluctuations in the solar wind at $\beta \sim 1$

Magnetic field



$Alfvenic\ turb$: $\omega \sim k_{||} v_A$

Density



MHD-scale turbulence

Kinetic-scale turbulence

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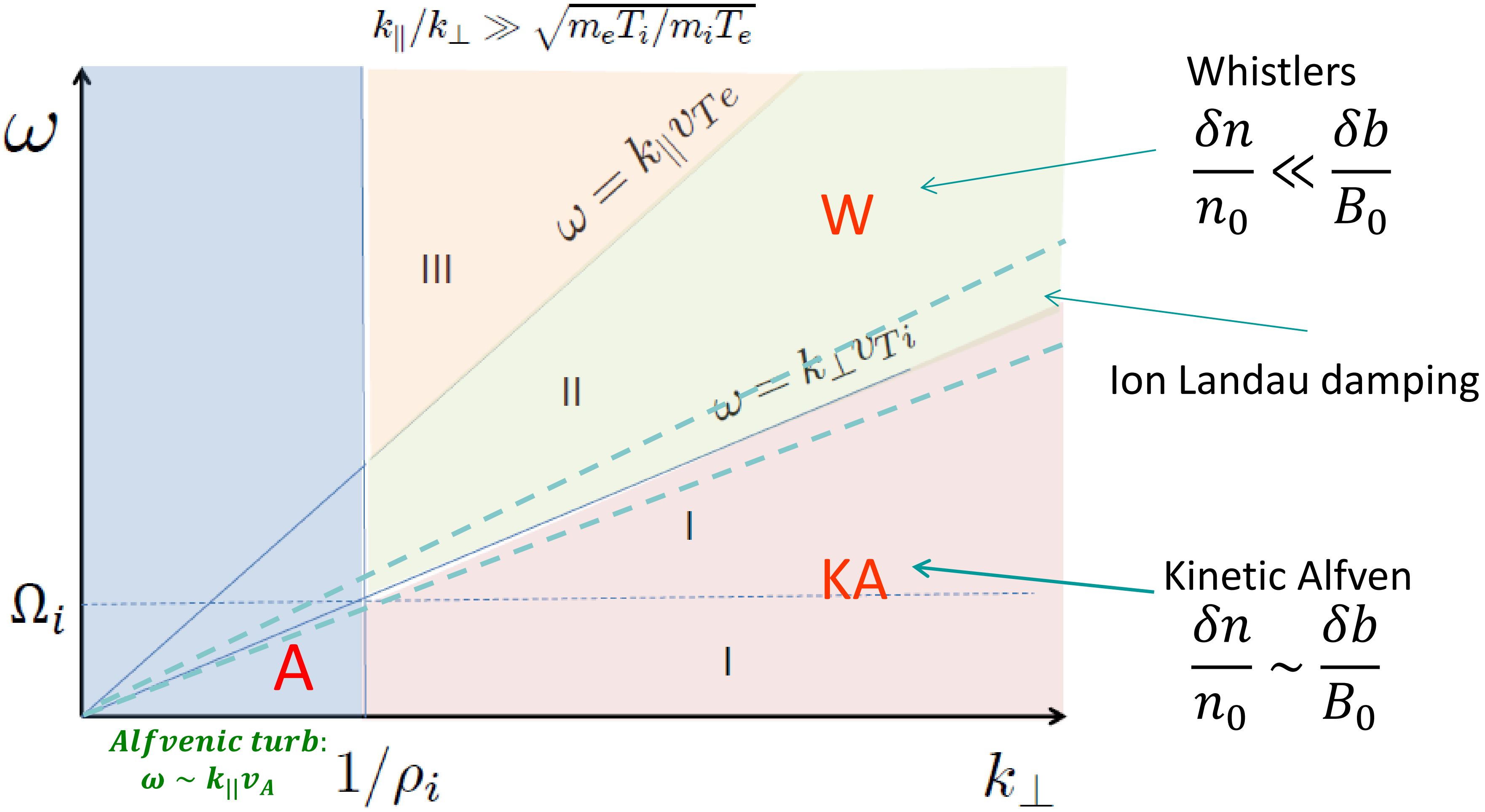
Mozer & Chen (2013) for electric field

Oblique waves in $\beta \sim 1$ plasma



Whistlers

$$\frac{\delta n}{n_0} \ll \frac{\delta b}{B_0}$$

Ion Landau damping

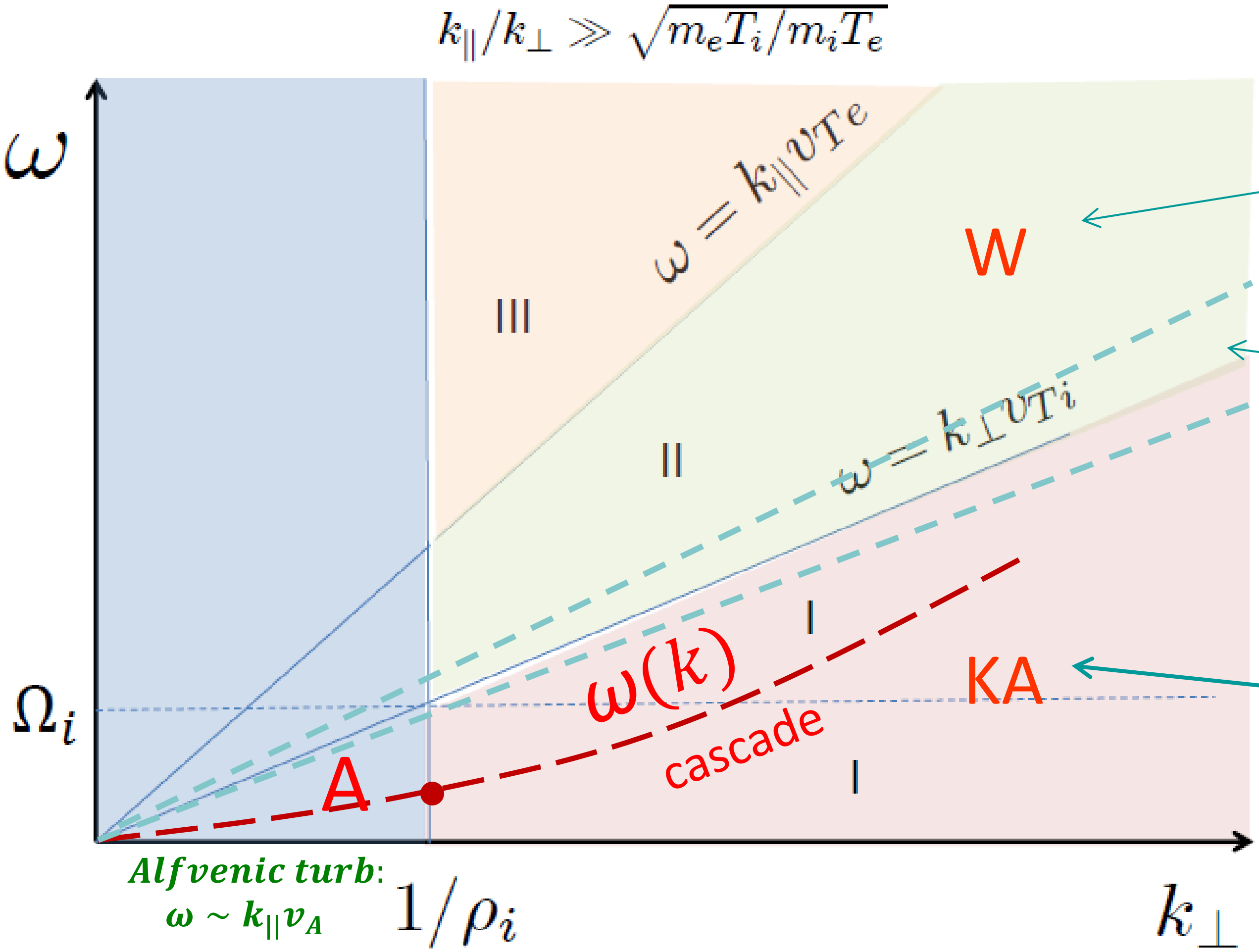
Kinetic Alfven

$$\frac{\delta n}{n_0} \sim \frac{\delta b}{B_0}$$

***Alfvenic turb*:**
$\boldsymbol{\omega \sim k_{\|} v_A}$

Howes et al ApJ (2006), SB, Horaites, Xia, Perez, ApJ (2013)
Chen, et al PRL (2013)

Oblique waves in $\beta \sim 1$ plasma



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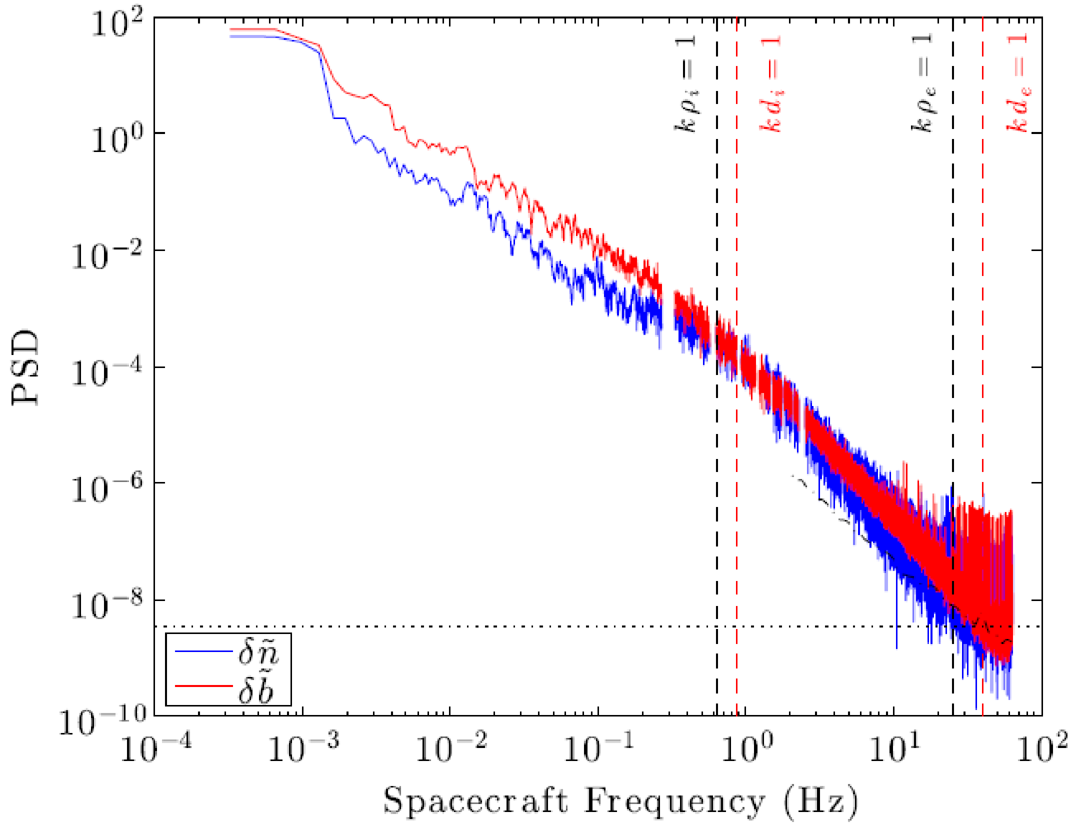
Chen, et al PRL (2013)

Sub-proton fluctuations in the solar wind

Magnetic field Density

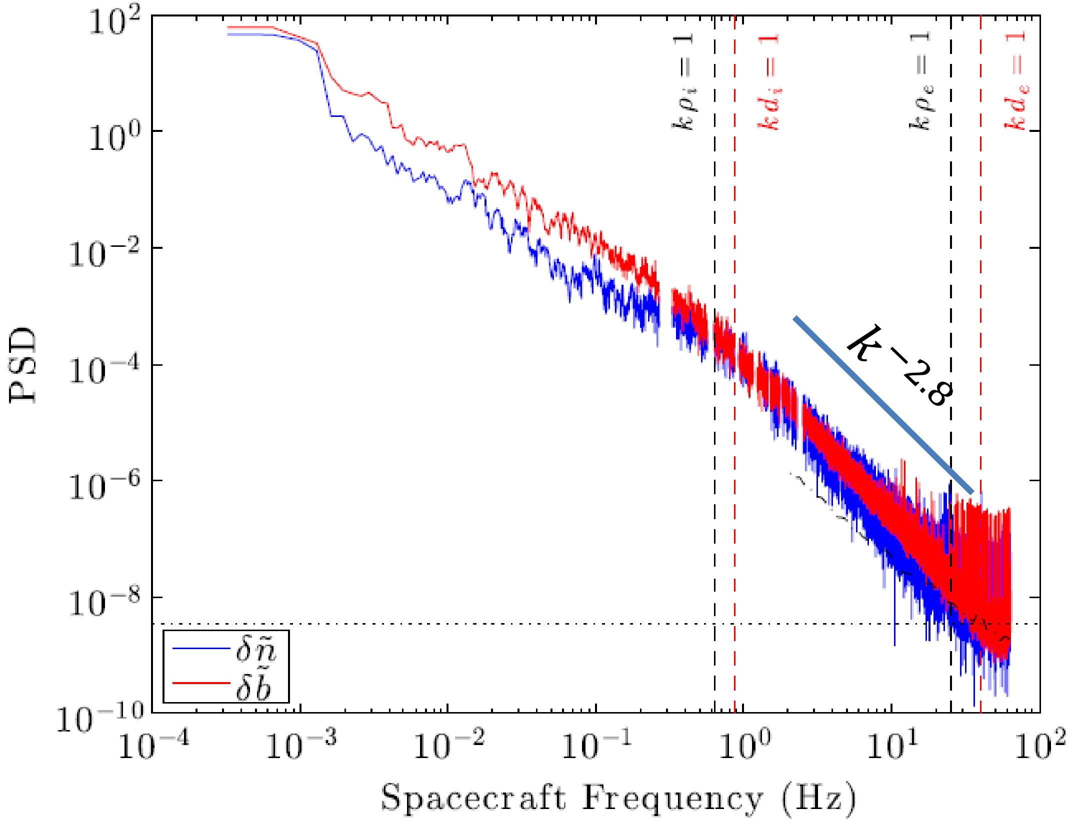


Near equipartition between b and n is more consistent with kinetic-Alfven modes
Chen et al PRL (2013)

Spectrum of sub-proton fluctuations

Magnetic field

Density

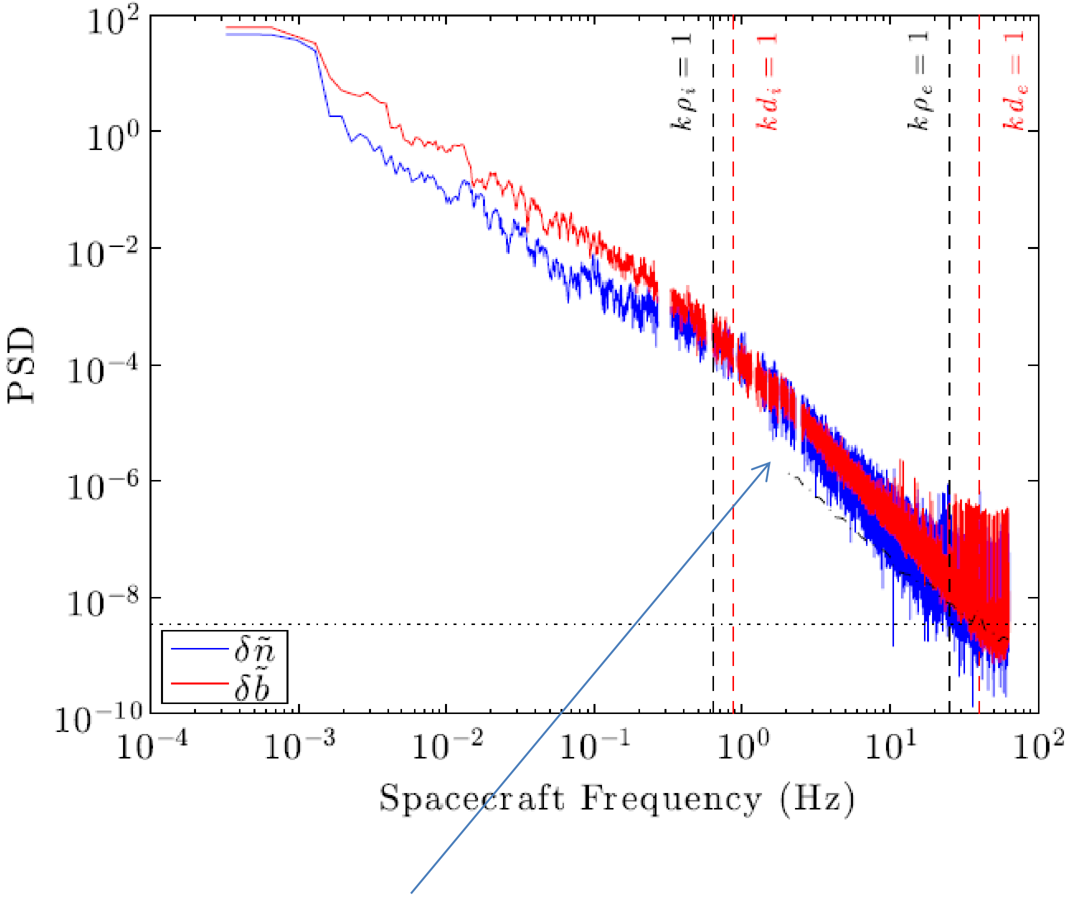


Kolmogorov-like arguments lead to -7/3 spectrum (e.g., Vainshtein 1973). The difference is not fully understood (Landau damping, Intermittecy, Non-gaussian distribution functions, etc?)

Spectral break of fluctuations in the solar wind

Magnetic field

Density



Spectral break corresponds to ion gyroscale in these observations.
Is this always the case?

Spectral break for $\beta \gg 1$ and $\beta \ll 1$

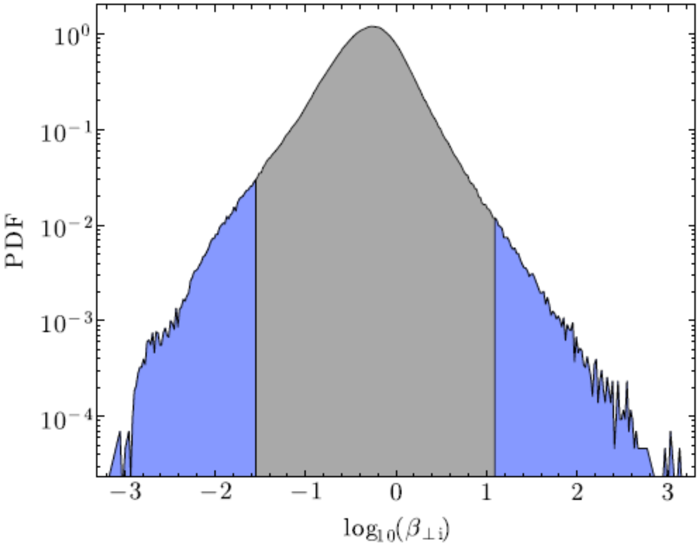


Figure 1. Probability density function (PDF) of $\log_{10}(\beta_{\perp i})$ in the solar wind. Intervals used in this letter are from the extreme parts of the distribution with $\beta_{\perp i} \ll 1$ and $\beta_{\perp i} \gg 1$ shaded in blue.

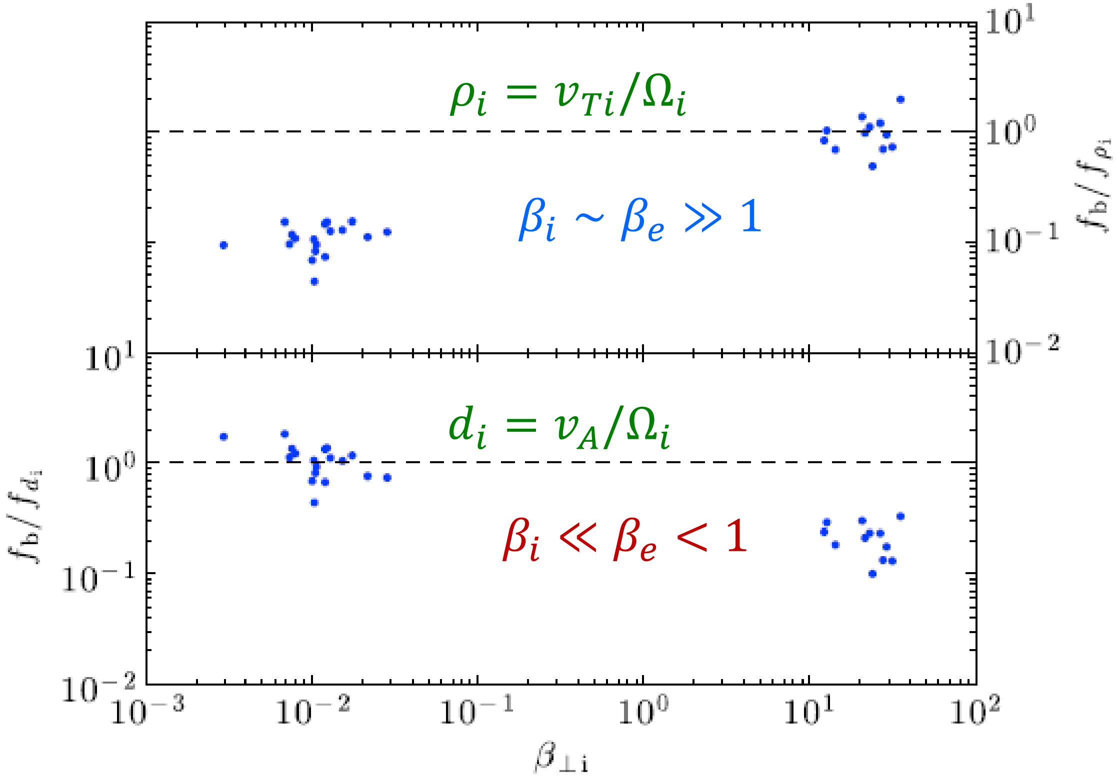
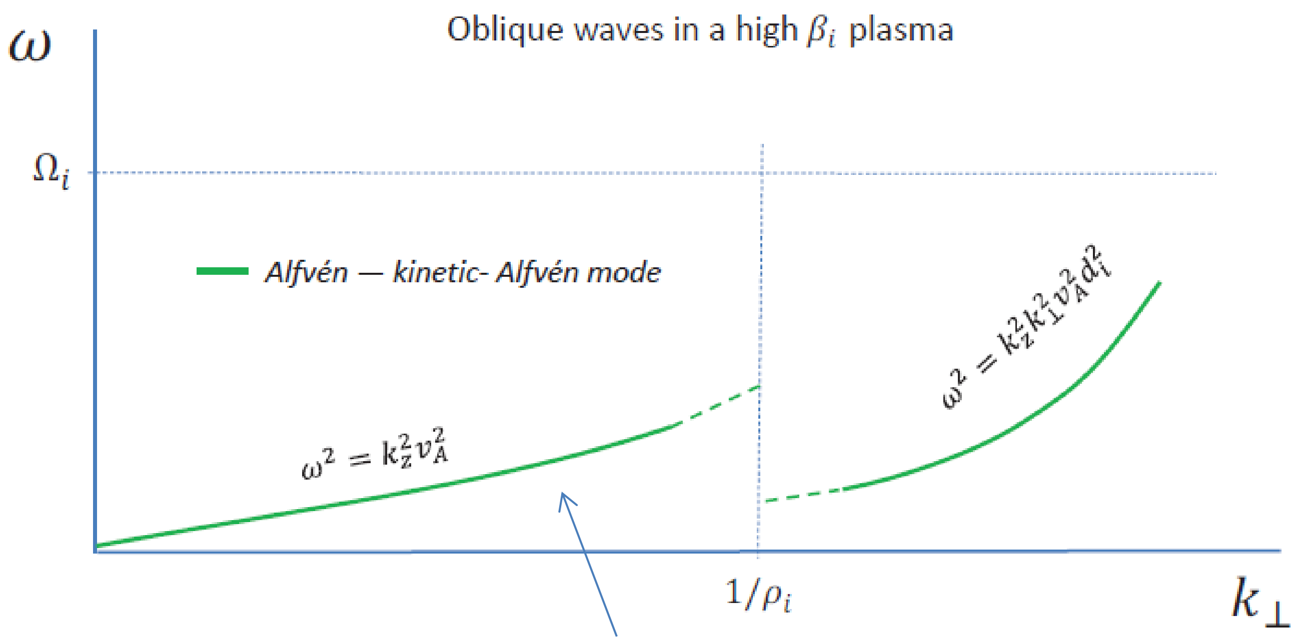


Figure 3. Ratio of measured break frequency f_b to the frequency corresponding to (top) $k\rho_i = 1$ and (bottom) $kd_i = 1$, as a function of $\beta_{\perp i}$ for all intervals in which a break was measurable.

$$\rho_i = v_{Ti}/\Omega_i$$
$$d_i = v_A/\Omega_i$$
$$\beta_i = \rho_i/d_i$$

Chen et al 2014

Alfven waves for $\beta \gg 1$

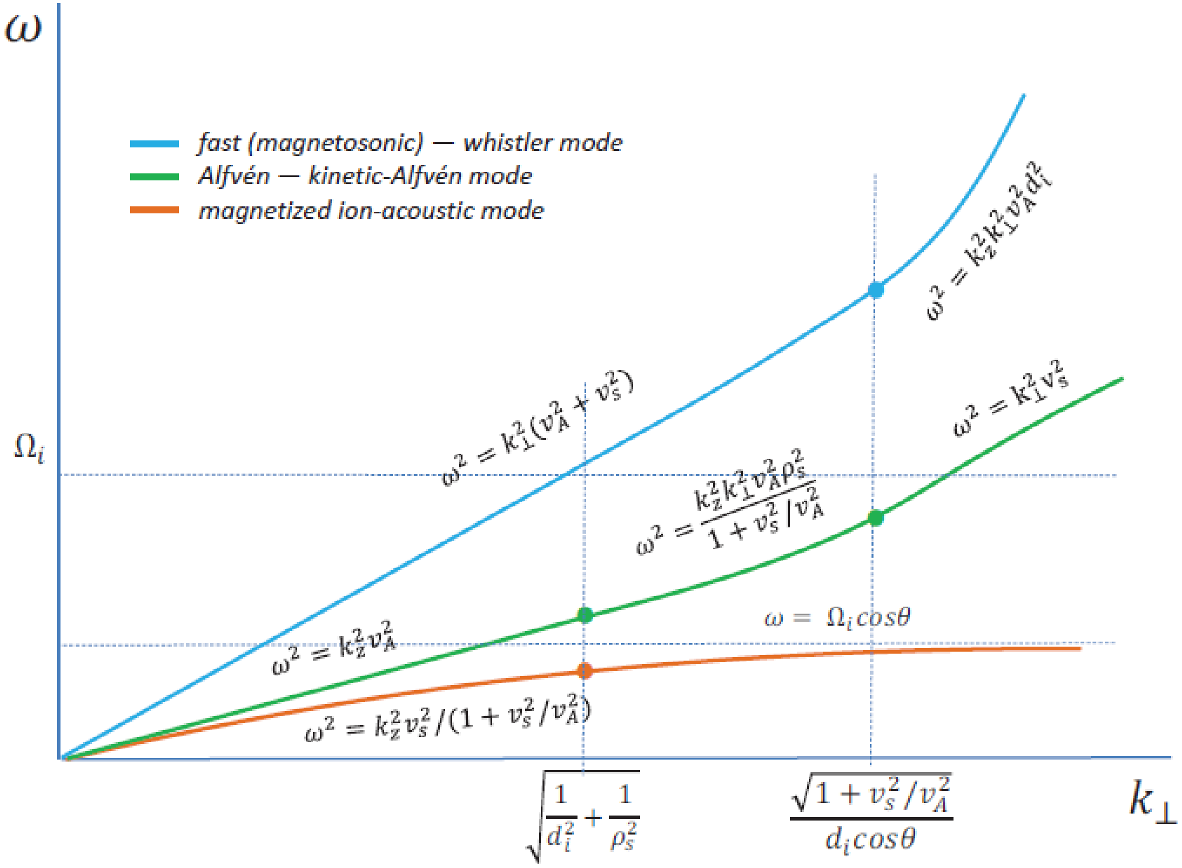


$$\omega^2 = k_z^2 v_A^2 \left[1 + \left(\frac{3}{4} + \frac{T_e}{T_i + T_e} \right) k_\perp^2 \rho_i^2 - 3k_z^2 \rho_i^2 - \frac{9}{8} \sqrt{\frac{\beta_i}{\pi}} k_\perp^2 \rho_i^2 \right]$$

The break is at $1/\rho_i$ - agrees with observations

SB, Chen, et al 2015

Alfven waves for $\beta_i \ll \beta_e < 1$



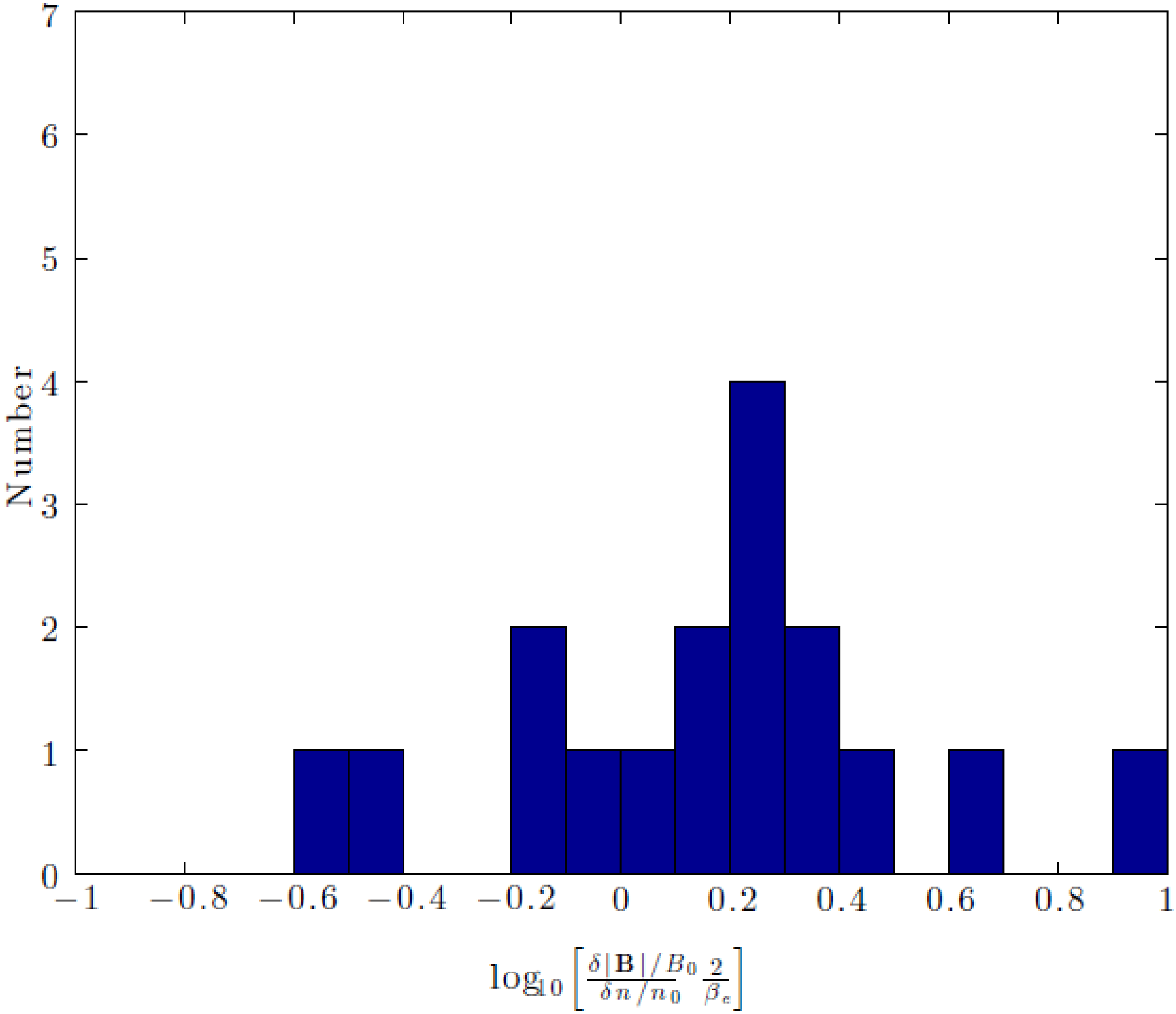
Oblique waves in a low β_i plasma

The break at $1/\rho_s$, does not agree with the observations

$$\rho_s = v_s/\Omega_i = \sqrt{T_e/T_i}\,\rho_i$$

SB, Chen, et al 2015

Pressure balance check



Fast modes:

$$\delta|B|/B_0 = \delta n/n_0$$

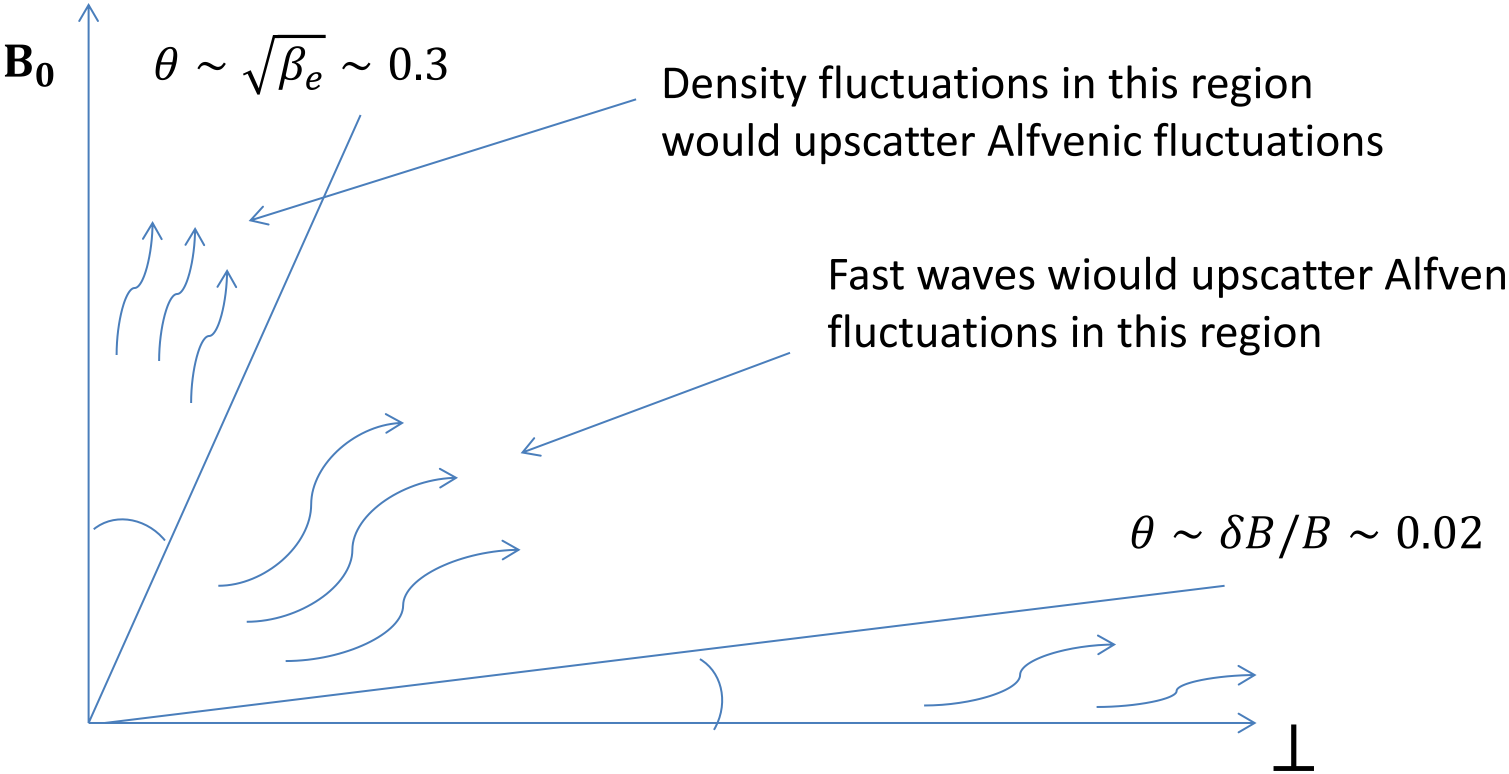
Ion-acoustic mode:

$$\delta|B|/B_0 = (k_\perp/k)^2(\beta_e/2)\delta n/n_0$$

Absencce of good pressure balance – Possible presence of fast or ion-acoustic modes

SB, Chen, et al 2015

Role of density fluctuations



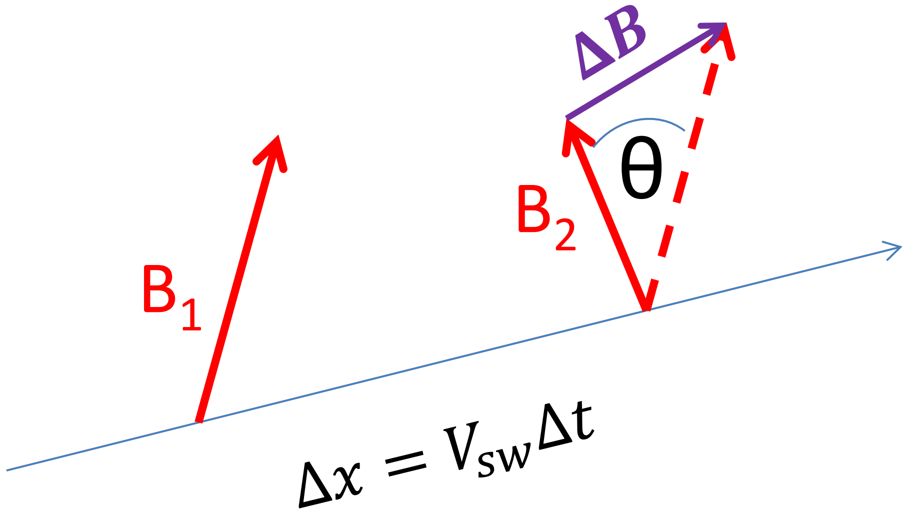
Density fluctuations in this region would upscatter Alfvenic fluctuations

Fast waves wiould upscatter Alfven fluctuations in this region

Processes of Alfven wave scattering by density fluctuations play important role

Field parallel turbulent cascade may be important (Roberts & Li 2015).

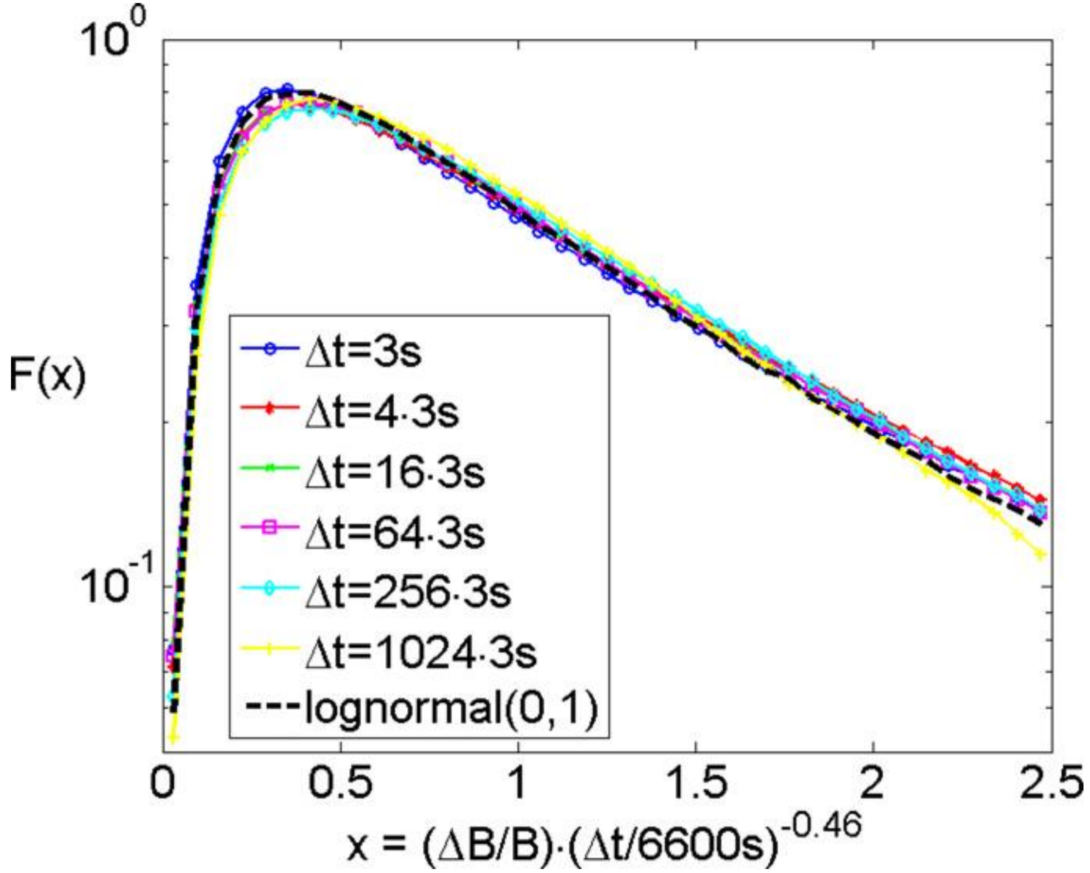
Magnetic field rotations



$$P\left(\frac{\Delta B}{B};\Delta t\right)=\left(\frac{\Delta t}{\Delta t_0}\right)^{-\alpha}F\left(\frac{\Delta B}{B}\left(\frac{\Delta t}{\Delta t_0}\right)^{-\alpha}\right)$$

Zhdankin et al (2012)

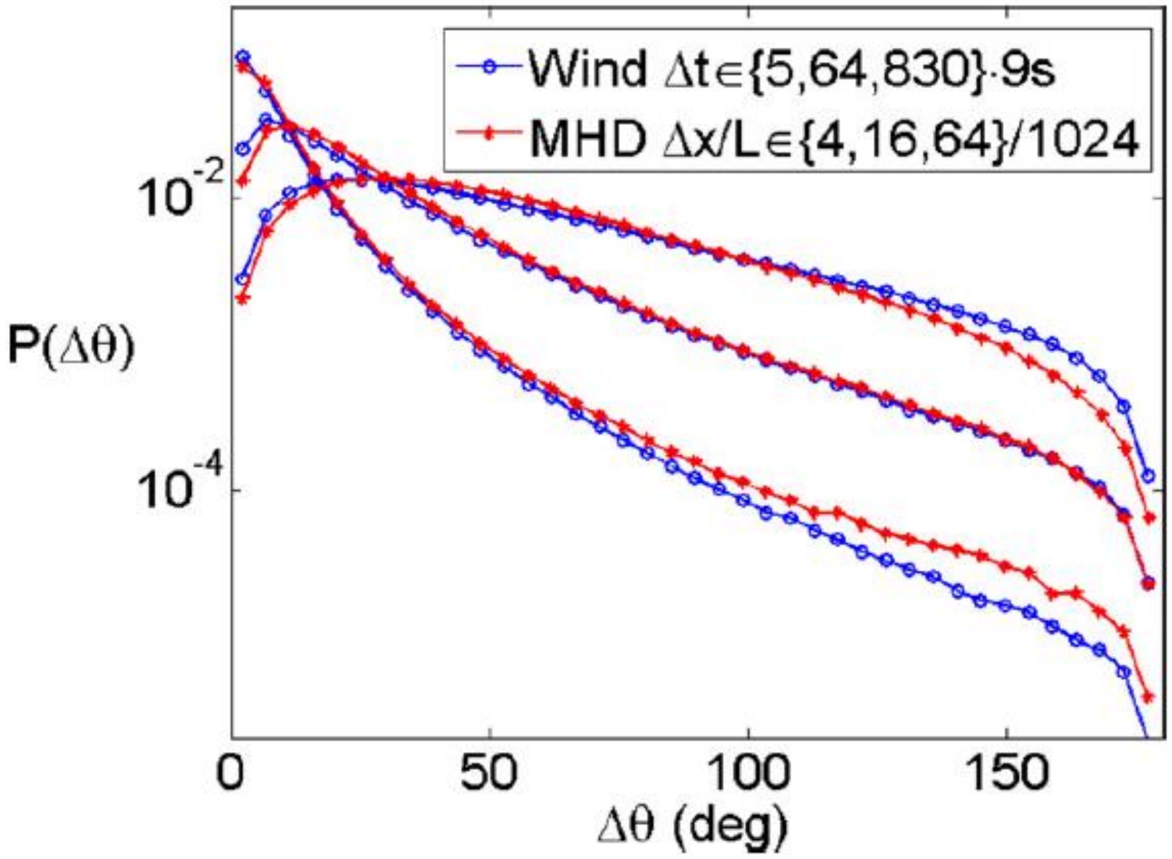
Magnetic field rotations in the solar wind



Zhdankin et al (2012): MHD
Chen et al (2015): Kinetic scales

$$F(x) = \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{1}{2}\log^2 x\right)$$

Magnetic field rotations in the solar wind and in numerical simulations



Zhdankin et al 2012

Note difference in the scaling!

Fundamental questions to answer

- Waves and Turbulence in regimes of high and low plasma beta (kinetic, magnetic parts, residual energy?) Some answers from future NASA missions?

- Origin and role of structures in solar wind turbulence (originated at the sun? Developed in the solar wind turbulence?)

- Role of kinetic effects, where they become important? Ions or electrons? Relevant for mechanisms of energy dissipation.

- Role of density inhomogeneities in a low beta plasma (scatter of Alfven waves; upscatter in energies?)

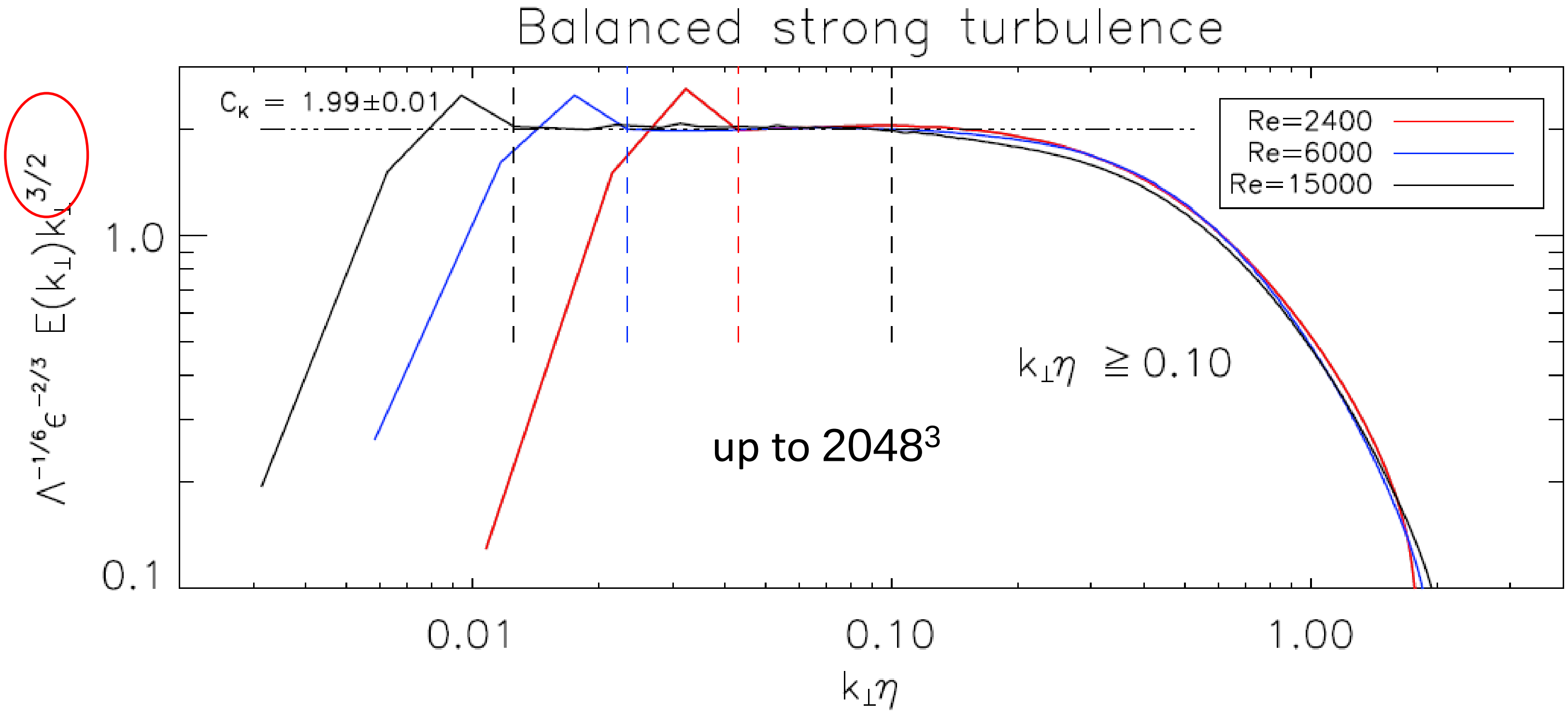
Contribution to the Discussion:

Comparison of numerical measurements of the MHD turbulence spectrum by Perez et al (2012, 2014) and Beresnyak (2011, 2012, 2014)

Perez et al (2014): arXiv:1409.8106

http://adsabs.harvard.edu/abs/2014arXiv1409.8106P

The -3/2 spectrum of strong MHD turbulence is obtained by Perez et al (2012)

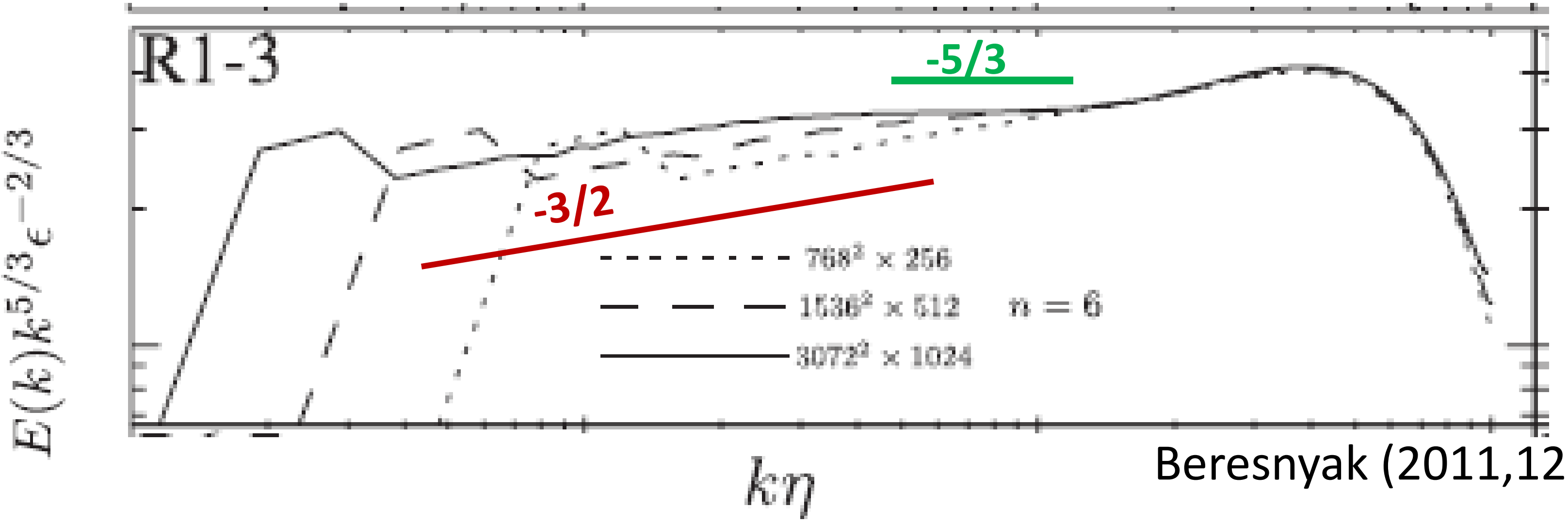


up to 2048^3

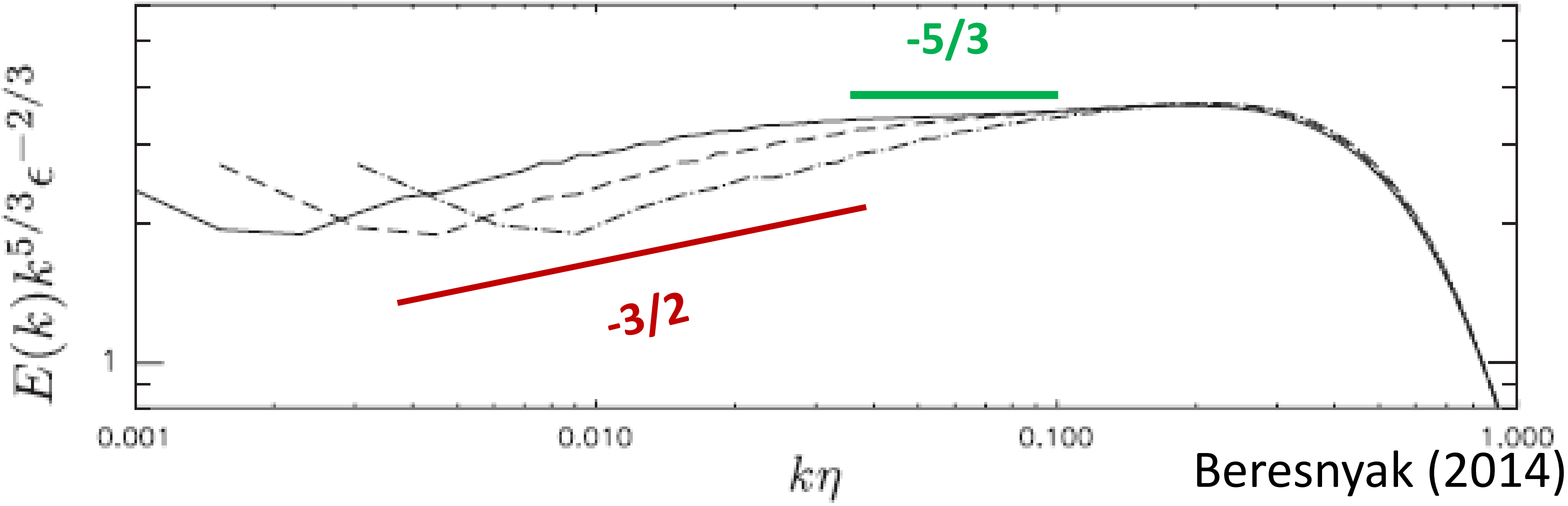
Computational resources: DoE 2010 INCITE,
Machine: Intrepid, IBM BG/P at Argonne Leadership Computing Facility

Perez et al, Phys Rev X (2012)

Beresnyak (2011, 2012, 2014)
concluded that the spectrum is -5/3

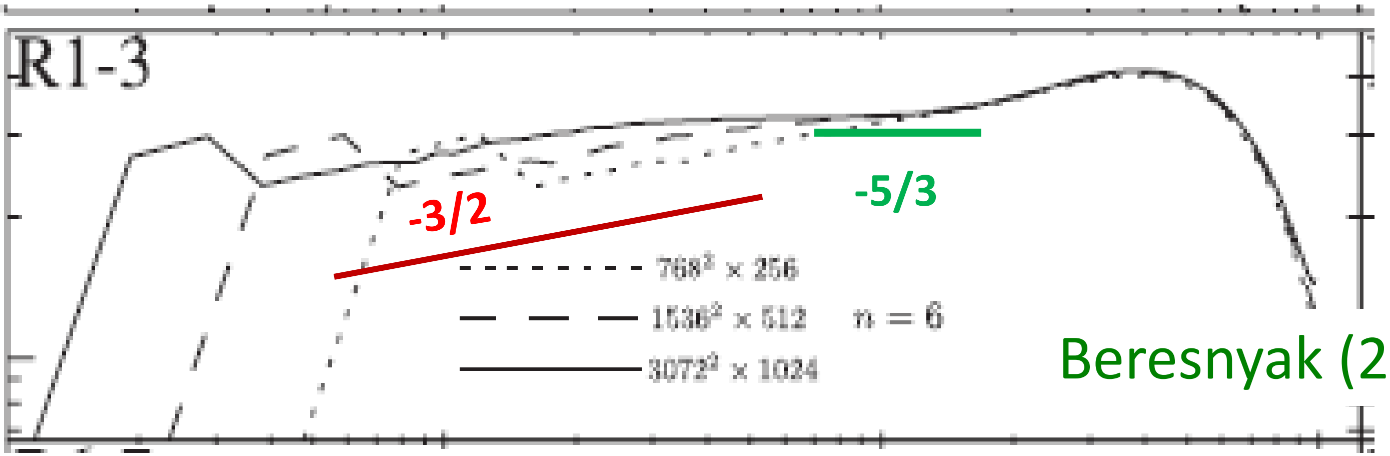


Beresnyak (2011,12)



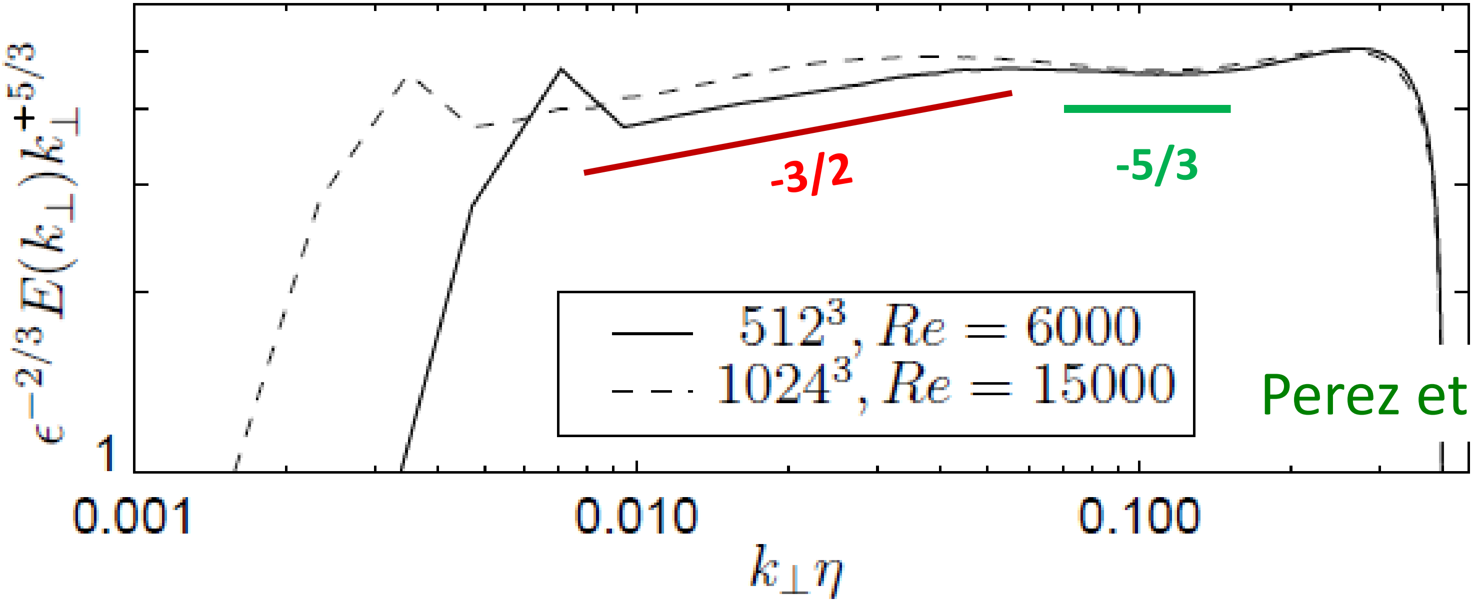
Beresnyak (2014)

Beresnyak's results are easily reproduced in unresolved runs



Beresnyak (2011,12)

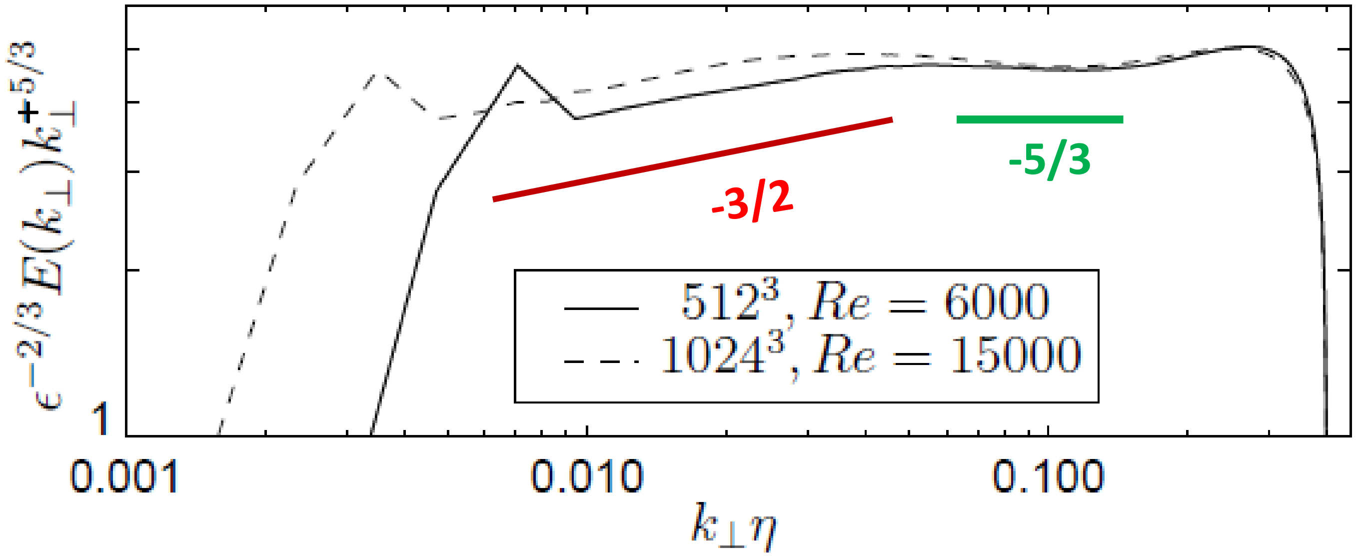
Unresolved MHD runs



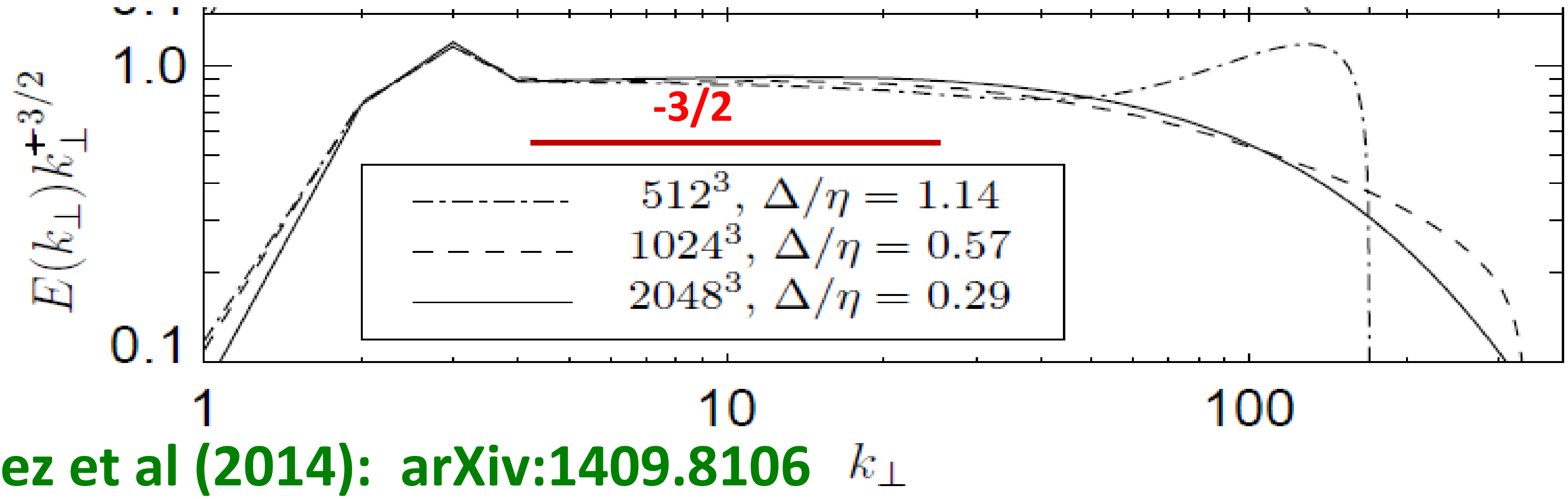
Perez et al (2014)

The «-5/3 spectrum» disappears in resolved runs

Unresolved MHD runs



The numerical distortion disappears as the resolution is increased

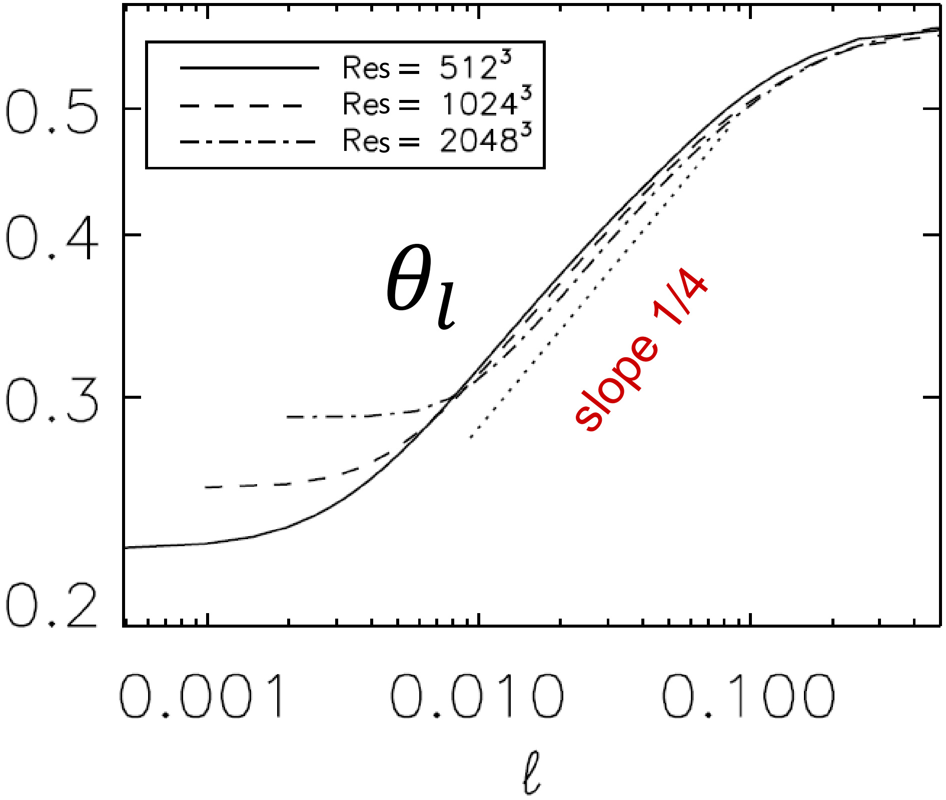


Perez et al (2014): arXiv:1409.8106

Dynamic alignment is well seen in resolved runs

$$S_{cross}(r) = \langle|\delta\tilde{\mathbf{v}}_r \times \delta\tilde{\mathbf{b}}_r|\rangle \qquad S_2(r) = \langle|\delta\tilde{\mathbf{v}}_r||\delta\tilde{\mathbf{b}}_r|\rangle$$

Alignment angle: $\theta_r \approx \sin(\theta_r) \equiv S_{cross}(r)/S_2(r)$



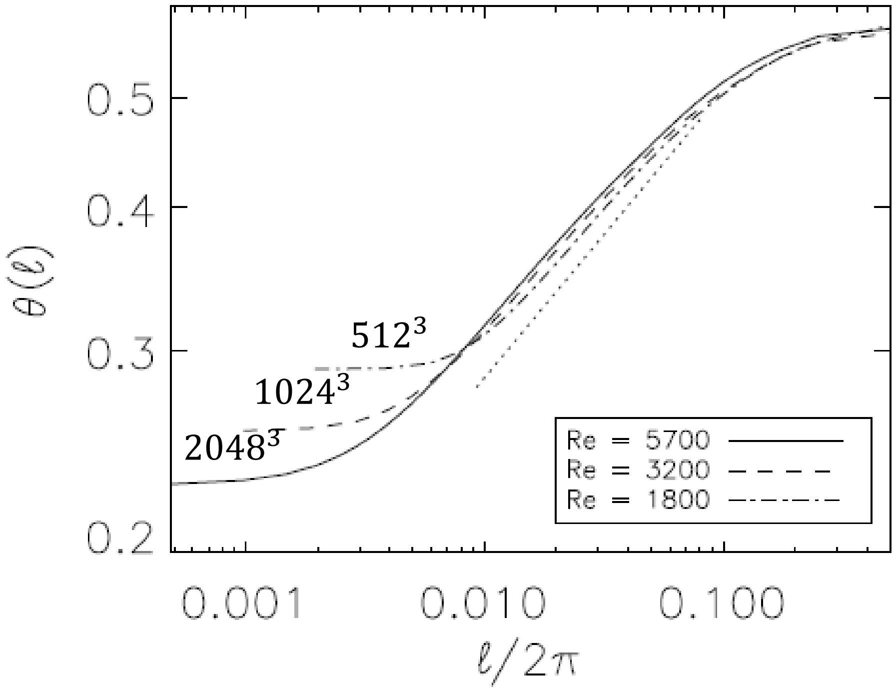
Magnetic and velocity fluctuations become progressively stronger aligned at smaller scales. Form sheet-like structures

$$\theta_l = (l/\Lambda)^{1/4}\theta_0$$

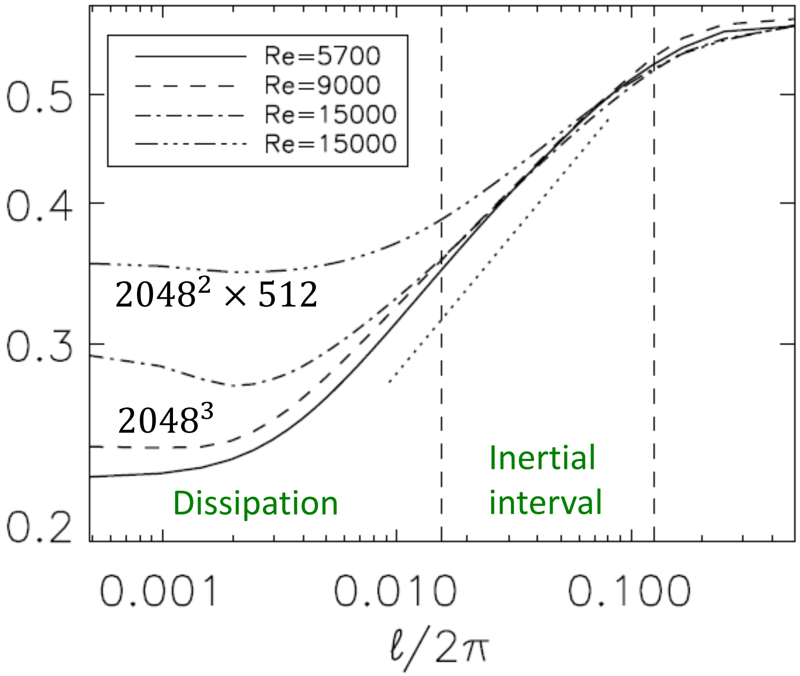
Mason et al 2011,
Perez et al 2012, 2014

The alignment is well seen in resolved numerical simulations!

Dynamic alignment is broken in unresolved runs



Alignment is preserved in well resolved simulations at all scales, even inside the dissipation range.



Alignment is broken in unresolved simulations

Mason et al 2011, Perez et al 2012, 2014

Conclusion

Numerical simulations by Beresnyak (2011, 2012, 2014) are drastically unresolved in the region where the «-5/3 spectrum» is observed.

The «-5/3 spectrum» is a purely numerical, unphysical effect; it disappears as the resolution is increased.

The spectrum is -3/2 in resolved simulations by Perez et al (2012, 2014).